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CONVENTION OR NECESSITY? THE IMPERSONAL IN MATHEMATICAL WRITING

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Why does most academic writing appear to exclude the voice of the author and the human side of the subject matter? This tendency to the impersonal is perhaps even more marked in mathematics than in other subject areas and seems to be one of the sources of disaffection for some students. But is it possible to produce more 'user-friendly' mathematical texts? Does the nature of mathematics itself constrain the choices available to writers or are the constraints only conventional? How does the personal narrative style of school 'investigations' relate to the genres used in more advanced mathematics? This paper makes a start at discussing these questions.

Introduction

A rectangle has equal diagonals.	If you measure the lengths of the diagonals of a rectangle, you will find that they are the same.	The measurements of the lengths of the diagonals of a rectangle are always equal.
(1)	(2)	(3)

Which of these statements is the 'most mathematical'? Most people I have asked (primary and secondary teachers and mathematics education researchers) have agreed that it is the first one, though some have tried to give me answers to other questions as well or instead, for example: which do you prefer? which is the easiest for children to understand? But what is it that is 'more mathematical' about statement (1)? What does it say that is different from what the other statements say? In each case 'the same content' is expressed – the general relationship between the lengths of the diagonals of a rectangle. There seem to me to be a number of areas of difference:

redundancy: We may assume that, within the context of the area of mathematics in which these statements appear to be located, the 'equality' of diagonals must relate to their length. Explicitly stating that the lengths of the diagonals are equal introduces redundant information and either violates the general principle of communicating only what is necessary or suggests that the author is insufficiently knowledgeable in the field to recognise the existence of this redundant information.

temporality: It is unnecessary to claim this is 'always' true because it is a fundamental assumption about mathematical facts that they are true for all time (and hence to make such a claim is to demonstrate an apparent lack of understanding of the nature of mathematics).

human activity: Mathematical facts are not dependent on empirical verification. To suggest that the equality of the diagonals is dependent on the result of the human activity of measuring thus again demonstrates a lack of understanding of the nature of mathematics[1].

Or does it? Is it a purely social phenomenon that we interpret texts in this way, seeing the statements (2) and (3) as examples of immaturity or lack of mathematical understanding? They are, after all, just as true as the first one (though the references to measurement might be challenged on the grounds that measurements are not going to be accurate enough to demonstrate equality). They can, rather than being seen as less mathematical, be seen as examples of different mathematical genres, expressing different aspects of mathematics for different purposes, for example, to instruct a student or to display the way in which the mathematician discovered the phenomenon. Why should we privilege the formal (non-redundant, timeless, non-human, context-independent) text as more mathematical?

The relationship between form and content

Although it is possible to say that all three of the statements above are in some sense 'about' the same mathematical relationship, the 'content' of each text is not independent of the form in which it is written. The grammar of the text is part of the system of choices (Halliday, 1985) available to the writer (consciously or unconsciously) to shape the nature of what they are trying to convey; it also forms part of the resources that the reader uses to make sense of the text. The choice between passive and active mood, for example, is not solely a choice about form but is also a choice about whether to represent or to obscure the role of the agent in the process. The grammatical metaphor of nominalisation (Halliday, 1998), using the nominal *rotation* rather than a phrase involving the verb *rotate*, is not simply a different way of expressing the same meaning but actually allows the writer to say new things such as *this set of rotations forms a group*.

Mathematics as an autonomous system *versus* Mathematics as a human activity

Does mathematics have an autonomous existence or is it the product of humans? Discussions of the philosophy of mathematics generally seem to suppose that these two views of mathematics are separate and incompatible. Yet even the greatest formalist does mathematics. Perhaps a more useful distinction would be between mathematical 'facts' and mathematical 'activity'. The conventional view of mathematical writing privileges the facts and obscures the activity. Solomon &

[1] I am using mainstream assumptions about 'the nature of mathematics' in order to explain our common responses to these statements, not because I am committed to such a philosophy of mathematics.

O'Neill's analysis (1998) of some of William Hamilton's writing about quaternions rests on this view. They use an extract from a letter published by Hamilton to illustrate what they describe as "two distinct component texts":

> *My train of thought was of this kind.* Since √-1 is in a certain well-known sense a line perpendicular to the line 1, *it seemed natural* that there should be some other imaginary to express a line perpendicular to both the former; and because the rotation from 1 to this also being doubled conducts to -1, it also ought to be a square root of negative unity, though not to be confounded with the former. Calling the old root, as the Germans often do, i, and the new one j, *I inquired what laws ought to be assumed* for multiplying together a + ib + jc and x + iy + jz. *It was natural to assume* the product = ax - by - cz + i(ay + bx) + j(az + cx) + ij(bz + cy); but what are we to do with ij?
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> (Hamilton, 1884, cited in Solomon & O'Neill, 1998, p. 214, my italics)

They identify a narrative "supertext" characterised by the use of the past tense and temporal ordering (in italics in the extract above) and a "mathematical sub-text" characterised by the "timeless present" and logical ordering, gaining its cohesion from the use of expressions such as *since* and *because*. The implication is that the narrative of Hamilton's processes of inquiry is not mathematics.

It is certainly possible to distinguish the two parts of the text but is it possible to interpret it differently? Is the narrative of Hamilton's thinking and feeling any less mathematical than the statements of his results? I would argue that you cannot separate the two parts of this text. This is not a mixing of two genres but is a single genre that presents a picture of the construction of mathematical knowledge involving human activity in speculating and reasoning – the logic is human logic as much as it is mathematical logic.

The stereotypical genre of the conventional present day journal text tends to lack the temporal narrative of inquiry but, nevertheless, still constructs a role for human beings. In particular, the use of imperatives, as Brian Rotman argues (1988), supposes a reader who engages with the text either as a "thinker", joining the author in constructing a mathematical world (by defining, naming, supposing), or as a "scribbler", performing the calculations that are necessary to complete the argument. Such texts use what Richards (1991) calls the logic of reconstruction rather than the logic of discovery. The mathematician's questionings and decision making are absent.

Solomon & O'Neill state that "mathematics cannot be narrative for it is structured around logical and not temporal relations" (p.217). As soon as you admit that mathematicians *do* mathematics, however, the doing must take place in time[2]. Rather than saying that it is only the timeless logic and the statement of eternal facts that is

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[2]I am, of course, perfectly aware that some would argue that mathematicians discover something that is independent of their doing. It must be clear by now that I position myself on the humanist side of the debates about the nature of mathematics, which have been presented much better than I can do elsewhere (e.g. Ernest, 1991; Hersch, 1998).

truly mathematical while Hamilton's personal narrative is extra-mathematical, it is possible to argue that the presentation of mathematics as an eternal and autonomous system actually misrepresents the nature of mathematics as the product of human activity.

Some present day examples

While historical examples are interesting in themselves and useful in that they can help us to question some of our assumptions, my primary interest lies in looking at the texts that are being constructed by present-day mathematicians. One clear result that has emerged from the analysis of mathematics research papers that Leone Burton and I have undertaken is that there is not a single way of writing mathematically but that there is enormous diversity (Burton & Morgan, forthcoming). I will discuss two examples of texts, extracted from published research papers, which construct rather different views of the place of the human mathematician within mathematics. In both cases, the extracts are taken from sections of each paper that are introducing the problems to be dealt with and presenting and giving reasons for the procedures to be used.

Extract 1

We shall consider the steady flow of a compressible viscous fluid above an infinite flat disc rotating with an angular velocity Ω about the z-axis. Thus, . . . the effects of streamline curvature and Coriolis force must be taken into account. The problem is formulated in terms of cylindrical polar coordinates to take advantage of the axisymmetric motion . . . Thus we introduce the non-dimensional coordinates and velocities (r, q, z) . . .

In this first example, the actions of the mathematician appear to be driven by the logic of the mathematics "Thus … the effects … must be taken into account", "Thus we introduce", and are further obscured by the passive "must be taken into account". Even the problem itself "is formulated … to take advantage", without acknowledging the human actions of posing problems and making judgements and decisions in the course of the problem solving.

Extract 2

We will now look more closely at the algorithmic aspects of Poincaré's Theorem. We wish to produce a mechanical procedure which . . .

What kind of mathematical model of a computing machine is necessary in order to carry out the procedure described in the preceding papers? . . . We need to be able to handle real numbers not as sequences of bits but as entities. We need to be able to compare two real numbers . . .

Let us go through the steps of the computation to see what kinds of operations are necessary. We need to start by making a decision as to how to represent the input data. . . .

In contrast, in this second extract the problems appear to arise from the mathematician's wishes and needs. The "decision as to how to represent the input data" is seen as a human decision rather than one that is determined solely by logic.

To what extent are these differences functions of the mathematical topics being addressed and to what extent is it a matter of choice? Could each text be rewritten in the style of the other? If this were done, what would change? Would it make any difference to the journal editors' decisions about suitability for publication?[3]

So what is the place of the personal in mathematical writing in school?

There are, of course, many differences between the texts of professional mathematicians and those produced by students of mathematics in school[4], not only in the topics addressed but also in the forms the writing takes. One reason for this is obviously because of differences in rhetorical function. The writing that school students do in mathematics classes generally serves or is intended to serve two kinds of functions. On the one hand, it is seen as part of the learning process (assisting reflection or recording for future reference) and, as such, is private – written by the student for herself. On the other hand (and sometimes at the same time) it is an important medium through which evaluation takes place and is thus addressed to a teacher or examiner. There is an interesting parallel with the writing of professional mathematicians here: for many mathematicians, much of their mathematical activity takes place through/in writing and the product of their activity is also judged through its presentation in writing. In the case of the mathematicians, it is clear that the two functions are served by very different genres of text – nobody would consider evaluating the value of mathematical research on the basis of the mathematician's rough workings. In school this distinction is not so clear cut.

In the UK context, students' 'processes' are subject to evaluation so the ways in which they represent their personal activity in the texts they produce is potentially significant to their success as students and to their future opportunities. The texts that students produce for GCSE coursework, for example, are expected to contain some form of narrative and a text that is written in an impersonal style may well be considered inappropriate by teacher-assessors (Morgan, 1996; 1998). It is clear that the necessity for students to write about their 'processes' (whether for learning or evaluation purposes) means that using a conventional impersonal academic style is

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[3]It is possible that well established and respected mathematicians are more likely than their more junior colleagues to write (or more likely to be published) in less conventional and more personal styles. There is some evidence to suggest that women mathematicians (often insecure in their employment and status) are less likely to depart from the conventional impersonal style (Burton & Morgan, forthcoming).

[4]I will only consider the writing done by students here because it represents the product of their mathematical activity, just as research papers represent the product of the activity of professional mathematicians (though the parallel is by no means exact). Similar questions could also be addressed for other types of school mathematics texts, including textbooks and teacher-produced writing.

not appropriate. The sort of style seen in Extract 2 above, however, might have something to offer.

My study of students' reports of their investigative work (Morgan, 1998) suggests that at least some, while attempting to follow their teacher's instruction to "write down what you did", fail to do so in a way that may be considered mathematical. Some included aspects of their work that were deemed inappropriate by teacher-assessors, including mention of social aspects of the work or their affective response to it. Others were so vague about their processes that they failed to communicate in a way that might be informative. For example, teachers responded negatively to use of the unelaborated statement "I found a formula".

Learning mathematics in school involves induction into mathematical discourses but the genres currently present in school mathematics classrooms are relatively limited (Mousley & Marks, 1991). In particular, there are few models of writing offered to students that combine the processes and the products of mathematical activity. Perhaps the narrative of Hamilton or the questioning of the author of Extract 2 would provide more appropriate models for teachers to use to support student writing than the conventional impersonal style they are more familiar with from their own induction into mathematics.

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