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METACOMMENTING: DEVELOPING ALGEBRAIC ACTIVITY IN A 'COMMUNITY OF INQUIRERS'

Alf Coles, Kingsfield School

Laurinda Brown, University of Bristol, Graduate School of Education

We are working on a Teacher Training Agency (TTA) funded project looking at the teaching and learning of algebra with one mixed ability year 7 class. We see ourselves as developing a 'community of practice' (Lave & Wenger, 1991) where the practice is not that of mathematician but of inquirer (Schoenfeld, 1996) into mathematics. The label articulated to the students for this practice was a 'purpose' (Brown & Coles, 1996, 1997) for the year of 'becoming a mathematician'. The teacher in this community acts as role model for inquirer and metacomments (Bateson, 1972) on the practice of inquiry. In this paper we will present evidence for how such metacommenting supports algebraic activity. We are also beginning to find evidence of the students metacommenting on the practice of the group.

Background

We are exploring the developing algebraic activity of one year 7 mixed ability group in a comprehensive school: Alf is the classroom teacher and Laurinda an intermittent visiting teacher-researcher. The research is a TTA funded project in which we are working on the challenge in Sutherland (1991): *Can we develop a school algebra culture in which pupils find a need for algebraic symbolism to express and explore their mathematical ideas?*

The definition of algebraic activity which we are using has three components: generational activities - *discovering algebraic expressions and equations;* transformational rule-based activities - *manipulating and simplifying algebraic expressions, solving equations, studying equivalence and form;* global, meta-level activities - *ideas of proof, mathematical structure, problem-solving. (This final component is not exclusive to algebra.)* (Kieran, quoted in Sutherland, 1997, p. 28).

Through trying to identify aspects of the third of these components both in our practice of mathematics and in that of the students, we have come to see algebraic activity as synonymous with thinking and working mathematically. We see our task in the classroom as developing a 'community of practice' (Lave and Wenger, 1991) where the 'practice' is not that of mathematician, since there may only be one of these (the teacher), but of inquirers (Schoenfeld, 1996) into mathematics. To do this we had to consider how the culture of the classroom, where algebraic activity is present all the time, would be established from the start of the academic year 1998/9 as the students began their life in a secondary school.

An account of these first lessons was presented at the last BSRLM meeting in Leeds (Coles and Brown, 1998) where we described the use

of the 'purpose' (Brown and Coles, 1996, 1997) articulated to the students of 'becoming a mathematician'. The students contributed their own meanings to 'becoming a mathematician' both implicitly (Claxton, 1996) and explicitly through writing about 'what I have learnt'. Alf's role was both to act as a role model of inquirer and to metacomment on the 'mathematical' behaviours of the students as they worked. In this paper we explore in more detail the meanings we give to the idea of metacommenting and present evidence for how such metacommenting by the teacher seems to be influencing the students' algebraic activity.

Metacommenting

Bateson (1972) says that 'human verbal communication can operate and always does operate at many contrasting levels of abstraction.' (p. 177). One of these levels he calls 'metacommunicative' which is when 'the subject of discourse is the relationship between the speakers.'(p. 178). We were interested in what the students did in relation to the task of 'becoming a mathematician' and needed to talk explicitly with the students about our relationship to the practice of inquiry into mathematics. To illustrate metacommunication Bateson (ibid) gave the following story:

What I encountered at the zoo was a phenomenon well known to everybody: I saw two young monkeys playing, i.e. engaged in an interactive sequence of which the unit actions or signals were similar to but not the same as combat. It was evident even to the human observer, that the sequence as a whole was not combat, and evident to the human observer that to the participant monkeys this was "not combat." Now, this phenomenon, play, could only occur if the participant organisms were capable of some degree of metacommunication, i.e. of exchanging signals which would carry the message "this is play."(Bateson, 1972, p. 179)

Obviously these messages are non-verbal and implicit and in fact Bateson comments that the 'vast majority of ... metacommunicative messages remain implicit' (p. 178). In establishing the culture in the classroom however we would need the messages to be explicit in the early days when the teacher comments to the students about their mathematical behaviours, because although we believe they are all naturally inquirers, at this stage they do not know the culture of doing mathematics which they are entering. We have come to call explicit metacommunicative messages metacomments.

How we work

Our methodology is situated within and draws upon what Bruner (1990) called a 'culturally sensitive psychology':

(which) is and must be based not only upon what people actually do but what they say they do and what they say caused them to do what they

did. It is also concerned with what people say others did and why ... how curious that there are so few studies that (ask): how does what one does reveal what one thinks and believes (p. 16-17).

In collecting data about the developing culture we concentrated on what Alf and the students did:

· From the start of the year Alf metacommented to the whole class describing his interpretation of what he saw students do in relation to the practice of becoming a mathematician eg *This group had an idea which they wrote down and tested and found it didn't work, so they changed their idea. That's a great example of what it is to think mathematically* (Research diary, September, 1998). Once a week Laurinda would observe the class and her focus of observation was metacomments.

· At the end of a sequence of lessons the students reflected and wrote on 'What have I learnt' which could be about content and also about 'becoming a mathematician'. Sometimes the students would write about something which Alf had explicitly stressed such as *I've learnt to look for patterns* and in other cases they developed their own statements *It's OK to make mistakes*. This sense of the students developing their own, several, meanings in relation to 'becoming a mathematician' and Alf not forcing a particular culture, simply staying with describing what was happening for himself or the students, was important to us to focus on the students' own need to use algebra.

We work within an enactivist methodology where we develop theories which are 'good enough for' (Reid, 1996) the data we are collecting, through interacting with our research diaries, videotapes from lessons, students' exercise books etc. The purpose of these theories is to transform our views in the act of staying with the detail of what the students do. We did not know when we started how the culture would develop in relation to Alf metacommenting on behaviours but what follows is a first attempt to show evidence for the developing algebraic activity of the students through this process. We are now starting to classify such behaviours and in turn are recognising other instances of them:

Strand 1 - students displaying behaviours implicitly and autonomously which have previously been metacommented on by the teacher.

Strand 2 - students displaying behaviours implicitly and autonomously which have been metacommented on by students in their writing.

Strand 3 - students explicitly metacommenting within the culture of the classroom.

This is very much work in progress but the episodes that we use below are chosen because we have seen similar evidence in other lessons and

they seem to us to stand for descriptions of algebraic activity of the students which are now part of the culture. This does not mean that each and every person in the class would exhibit every such behaviour but that we are 'coemerging' (Reid, 1996, p. 203) with these behaviours as part of the culture.

We are aware that the majority of the behaviours above will become implicit but we are interested in investigating whether it is possible to establish a community of inquirers in this way, through which the learning of mathematics is facilitated in a holistic manner, underpinned by algebra.

Episode One: The following interchange occurred during a lesson when the class was looking at drawing graphs of 'rules' such as times by three. A confusion had arisen about the difference between squaring and multiplying by two and between cubing and multiplying by three. The notation of powers for squaring and cubing had been discussed briefly in the previous lesson.

Claire: If you had N, N four, then you just have to put N times N times N times N.

AC: Exactly but that would be different from timesing it by four, those are different rules.

Claire: Because you can, if you write it out like that, you can have, there's a pattern because it's got two and then it's got three then it's got four and it keeps going up like a pattern. (Transcribed from videotape, December 1998)

'Noticing pattern' is a behaviour Alf had metacommented on from the first lesson, as being a part of what it is to think mathematically. In this incident Claire is both in the action of extending the pattern (Strand 1) and able to comment on it (Strand 3). There is also evidence of two of Kieran's (quoted in Sutherland, 1997) components of algebraic activity. The sense of spotting pattern and being able to continue a pattern is generational however what Claire is actually talking about is form and equivalence, which places her activity as transformational. In a lesson in January there was a strikingly similar incident in which Claire again built a sequence from the link between 3N and N add N add N, etc., which led to aN = N + N + ... + N (a times). On the baseline tests at the start of the year she came out as one of the weakest in the group and is on the school register of students with special educational needs, however there seems to be evidence that 'noticing pattern' is now a part of her, a part of what she sees mathematics lessons being about and she spontaneously generated the higher powers of N, seen in this culture for the first time.

Episode Two: In working on the 'handshakes' problem the class came up against the problem of how can you find a quick way of working out 1+2+3 ... +14, the number of handshakes for 15 people. A student had offered the image: 14+13+12+11+10+9+8
1+ 2+ 3+ 4+ 5+ 6+ 7

Part of the conversation was as follows:

- 15 goes to 15 times 14 divided by two.

- This is double.

- Same as fifteen times seven.

- Equals one hundred and five.

Alex: It all equals fifteen so 'cos there's fourteen numbers you times and then halve.

Ben: That's the same as what Paul said and then halve it.

Chloe: I've got something that I think will help. Can I come to the board? ... You could do it like this: fifteen add fifteen add fifteen add fifteen add fifteen add fifteen add fifteen, but times is quicker, it's fifteen times seven.

Both Alf and Laurinda, as teachers, were confused during the start of this discussion as were other students. The culture, however, supports meaning-making for all participants and since there was a lot of energy to try and sort this out, Alf let the discussion run on. The students were genuinely inquiring into this problem and offering their different ways of seeing it. Chloe's comment shows that she sees supporting others as being part of the culture of the classroom and this is in fact something she has explicitly written about before (Strand 2): *I think we have been working as mathematicians as a class because we have been sharing our problems and solving each others problems*. Here she is implicitly offering a structural insight which is evidence of Strand 1 and is working at a global meta-level algebraically as she works on how to convince the others in the class.

Episode Three: After we had worked for some time on the problem in episode two Ben asked:

Ben: Is there any way you could do N in this?

This comment took the discussion forward and is Strand 3 because Ben sees that there might be the possibility of going to the general structure and is able to comment on this explicitly. There is also some evidence of Ben seeing the need for algebra.

Conclusion

We conclude with the feedback we got from our session on the ideas in this paper, at the BSRLM conference. We asked participants to view the

video excerpt from which episode one is a transcript, using either the algebra definition or our metacommenting strands as a frame. We were interested in whether anyone else could use our frames and if not, why not? The use of the algebra definition to classify behaviour provoked much discussion. In contrast, the use of our metacommenting strands was problematic since it became apparent that it required a knowledge of the history of what had happened in the group. This does not invalidate our strands for interpretation but simply underlines the need for researchers in this type of study to live the experience of the group over time.

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References

Bateson, G, 1972, *Steps to an Ecology of Mind*, New York: Chandler

Brown, L and Coles, A, 1996, 'The Story of Silence: Teacher as Researcher, Researcher as Teacher'. In Puig, L and Gutiérrez, A (Eds), *Proceedings of the Twentieth Annual Conference of the International Group for the Psychology of Mathematics Education*, Vol 2 pp 145-152. Valencia

Brown, L (with Coles, A), 1997, 'Being True to Ourselves, Teacher as Researcher: Researcher as Teacher'. In Zack, V, Mousley, J and Breen, C (Eds), *Developing Practice: Teachers' Inquiry and Educational Change* (pp.103-111). Victoria, Australia: Deakin University

Bruner, J, 1990, *Acts of Meaning*, London: Harvard University Press

Coles, A and Brown, L, 1998, 'Developing Algebra - early evidence from a case study looking at the beginning of year 7'. In *Proceedings of the day conference of BSRLM, Leeds*, November (in press)

Claxton, G, 1996, 'Implicit Theories of Learning'. In Claxton, G, Atkinson, A, Osborn, M and Wallace, M (Eds), *Liberating the Learner* (pp. 45-56). London: Routledge

Lave, J and Wenger, E, 1991, *Situated Learning: Legitimate Peripheral Participation*, CUP, Cambridge

Reid, D A, 1996, 'Enactivism as a Methodology'. In Puig, L and Gutiérrez, A (Eds), Proceedings of the Twentieth Annual Conference of the International Group for the Psychology of Mathematics Education, Vol 4 pp 203-209. Valencia

Schoenfeld, A H, 1996, 'In Fostering Communities of Inquiry, Must It Matter That The Teacher Knows "The Answer"?', *For The Learning of Mathematics*, Vol 16, 3 pp 11-16

Sutherland, R. 1997, *Teaching and Learning Algebra pre-19*, London: RS/JMC