A CRITIQUE OF RELATIVISM IN MATHEMATICS EDUCATION: THE NEED FOR AN *OBJECTIVIST* PERSPECTIVE IF WE ARE TO FACILITATE COGNITIVE GROWTH

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Many constructivists tag as 'absolutist' references to mathematics as a body of knowledge, and stake-out the moral high-ground with the argument that mathematics is not only utilised oppressively but that it is, in-itself, oppressive. With much reference to Paul Ernest's (1991) Philosophy of Mathematics Education this tag has been justified on the grounds that ifmathematics is a social-cultural creation that is mutable and fallible then it must be social acceptance that confers the objectivity of mathematics. I will argue that mathematics is a body of knowledge the objectivity of which is independent of origin or social acceptance. Recently, Paul Ernest (1998) *has attempted to include the category of logical necessity in his elaboration of the objectivity of mathematics. I will argue that this inclusion of logical necessity not only represents a V-turn, but that the way in which Ernest has included this category is an attempt to maintain his earlier position that it is social acceptance that confers the objectivity of mathematics.*

Introduction

There is much in the literature of mathematics education that downplays mathematics as an abstract body of knowledge that has developed over the centuries and to which schoolchildren are to be encultured (Bereiter 1994). For example, if a syllabus requires Pythagoras' theorem to be known by students then absolutism (that mathematics is immutable and infallible) is sometimes seen to be implied (Jaworski 1994), and Therlfall (1996) argues that any reference to mathematical concepts is 'absolutist' and does not enable us to characterise a child's thinking. This is a form of *relativism* that downplays mathematics as an abstract body of knowledge and emphasises pupil learning at the expense of the subject to be learnt.

There are many who stake-out the moral high-ground of this pedagogical relativism by arguing that mathematics is not only utili sed oppressively but that mathematics is, *in itself*, oppressive. For example, Borba & Skovsmose (1997) argue that it is not only the case that *[m]athematics becomes part of the language ofpower* (p.17), but that in modelling reality *we are placed in a magical world where the grammar of mathematics fits the Platonic world which we are talking about. Here we have the necessary information; we calculate; and the calculation becomes either right or wrong* (p.18, emphasis given). The authors argue that the certainty of mathematics is a myth, and oppressive establishments confer this myth onto their arguments by expressing their arguments in a mathematical form. Seemingly, for the authors, it is not so much the model or the set assumptions that is contentious (oppressive, ideological, based on myth, etc.), but the 'magical Platonic world' which lies at the heart of the problem. The authors state that if teachers studied the fallibilistic approach to mathematics as advocated by Ernest (1991) *or change the structures of curricula to incorporate project work, or emphasise the need of students to choose their own problems as the basis for modelling situations, then we could challenge this ideology of certainty* (p.19). Read Ernest, teach mathematics that is 'relevant' , context-specific and in accordance with the needs and aspirations of the pupils, and the 'ideology of certainty' can be challenged and society might be a better place to live. The fundamental error in this argument is that it is the abstract nature of mathematics that is responsible for ideology and oppression, and this argument is diversary in that it shifts attention from the real causes of ideology and oppression (see Matthews 1980). Many mathematics educationalists commit this error. For Taylor (1996): *[m]odern school mathematics continues to be influenced strongly by the rationalist myth of cold reason (p.163) ..... What is needed, therefore, is a referent for epistemological reform that empowers teachers to engage their students in discursive practices that emancipate them from the indoctrinating and dehumanising influence of repressive cultural myths* (p.167). Even rationality itself has becomes a 'male-myth' (e.g. Walkerdine 1994, Ernest 1998). Despite all good intentions, mathematics educationalists themselves generate myth by attributing myth and oppression to mathematics as an abstract body of knowledge, and despite their 'political correctness', their arguments could disempower us as to what myth, ideology and oppression really is. This paper will employ rationality (what is wrong in being masculine?) to show that much of the nonsense that is written in mathematics education rests on the mistaken inference that if the certainty of mathematics is a myth then the emphasis should be placed on student learning rather than the content (the body of know ledge) to be taught.

Much reference has been made to Paul Ernest's (1991) *Philosophy of Mathematics Education,* Ernest (1998) has recently expressed social constructivism as a philosophy of mathematics and speaks on behalf of social constructivism. For these reasons Ernest will be centrally 'critiqued' in the next section to show the philosophical implications of the relativism (the downplay of mathematics as a body of knowledge) that exists within mathematics education. The subsequent section is the conclusion which spells out the pedagogical implications of this relativism.

What Possible Sense can we make of Paul Ernest's Relativism?

In *The Philosophy of Mathematics Education,* Paul Ernest (1991) at length shows that mathematics is mutable, fallible, corrigible, and open to revision. He argues that mathematics does not have the certainty as was once thought because of the contradictions that exist amongst its theorems:

> *We have seen that a number of absolutist philosophies of mathematics have failed to establish the logical necessity of mathematical knowledge* (p. 13).

So what does give mathematics its' 'seemingly' objective certainty? Ernest (1991) answers that it is social acceptance:

> *Objectivity itself will be understood to be social* (p. *42) .... Publication is necessary (but not sufficient) for subjective knowledge to become objective mathematical knowledge* (p. *43) .... To identify the immutable and enduring objectivity of the objects and truths of mathematics with something as mutable and arbitrary as socially accepted knowledge does, initially, seem problematic. However we have already established that all mathematical knowledge is fallible and mutable. Thus many of the traditional attributes of objectivity, such as its enduring and immutable nature, are already dismissed. With them go many of the traditional arguments for objectivity as a super-human ideal. Following Bloor we shall adopt a necessary condition for objectivity, social acceptance, to be its sufficient condition as well* (p. 45).

Ernest's position is that mathematics is objective because it is socially accepted, and if this is problematic because socially accepted knowledge is mutable and fallible, well so then is mathematics. This position is *interesting* (as defined by Phillips 1998) because any argument that defends such a controversial position must be interesting. The objectivity of mathematics cannot be reduced to consensus - the theorem of Pythagoras is true, and can be proven true for all rightangled triangles independently as to whether or not it is accepted true by the academic community. Consider the computer 'proof' of the 4-colour theorem: no one has checked the 'proof' to see if the theorem has been actually proven; but suppose an able mathematician was granted thousands of years to go through each line of working of the proof, and supposing the mathematician found an error in the proof (and generated a number of papers, hence gaining a consensus), this error would have been committed prior to discovery by the mathematician or *acceptance* by the community (Rowlands *et alI997).*

It is one thing to argue that mathematics exists within some eternal Platonic realm of preexisting structures, but it is quite another to argue that it exists only within intersubjective agreement. I would argue that mathematics can best be seen as a *practice* that creates *objective problem situations* (as defined by Chalmers 1982) the existence of which is independent of the cognition of the individual mathematician and of the acceptance by the academic community. Once a problem situation has been created then the solution *exists* including all the various routes to the solution - mathematics is discovered, but not until the (mathematical) conditions have been developed that would allow the discovery to be made. For example, there is evidence to show that Early Bronze Age people were familiar with what we would refer to as Pythagorean triplets (Wood 1980). Historically, once the right-angled triangle comes into being then the relationship between the three sides (what we refer to as Pythagoras's theorem) comes into being. It is in this sense that Pythagoras's theorem was in existence prior to Pythagoras. Although Bronze Age people were aware of certain triplets such as the 3-4-5 and the 12-35-37 triangles (see Wood 1980), nevertheless Euclidean axioms had to be assumed before it took the genius of Pythagoras and his community to discover the existence of the relationship and to prove that the relationship exists for all right-angled triangles.

Seven years later, and Ernest (1998), in his book *Social Constructivism as a Philosophy of Mathematics,* is aware of the difficulties in sustaining an interesting position but seems to deny adopting such a position in the first place: *This position* (multiplistic relativism) *sometimes figures in 'knockdown' critiques of relativism. It is a weak and possibly insupportable 'straw person' of a position and certainly does not represent the epistemological relativism adopted by social constructivism.* P.249). That mathematics is objective because it is socially accepted is a form of mutiplistic relativism because the reduction of objectivity to consensus omits any criteria by which conflicting claims can be adjudicated. Ernest appears to be aware of just how interesting his position is and the need to cover his back by adopting *logical necessity* in his elaboration of the objectivity of mathematics:

> *Thus to participate in mathematical language games involves granting the necessity of some assumptions. Within that game, certain conclusions are therefore necessary* (p. 146).

This is a *V-turn!* What absolutist philosophies failed to establish in Ernest (1991), namely logical necessity *(We have seen that a number of absolutist philosophers of mathematics have failed to establish the logical necessity of mathematical knowledge* p.13) he *assumes* in Ernest (1998): *[1]hat the adoption of certain rules of reasoning and consistency in mathematics mean that much of mathematics follows, without further choice or accident, by logical necessity* (p. 248). It would seem as though Ernest has now adopted a *boring position* (as defined by Phillips 1998): *boring,* because if logical necessity was assumed in the first place then his position would not have been so controversial or *interesting.* Of course, we are all entitled to change our position, and over a seven year stretch it would be understandable if our position did change, but Ernest (1998) emphatically states in the introduction that his present work is an elaboration and extension of his earlier work. In shifting from an interesting to a boring position but not acknowledging the shift, Ernest (1998) admits to logical necessity on the one hand (logically he has to) but downplays logical necessity (on p.88 'logical necessity' is something of an act) in the attempt to maintain his earlier position:

> *Thus mathematical knowledge and the standards of proof that support it depend what the mathematicians of the day accept ....However this is not to say that mathematical knowledge (or the standards of proof) are arbitrary, irrational, or illogical. On the contrary, mathematicians accept as mathematical knowledge only that which stands up to rational public criticism and scrutiny, based on their best professional judgement and some explicitly stated rules* (p. 46).

> *In the spirit of these proposals, the social constructivist philosophy of mathematics takes 'objective knowledge' in mathematics to be that which is accepted as legitimately warranted by the mathematical community* (p. 147).

According to Ernest, objective knowledge is based on professional judgement and is *that* which is accepted. Despite the inclusion of logical necessity, we have here a consensual view of the objectivity of mathematics. However, what if the mathematics community degenerates, would *that which is accepted* be objective mathematical knowledge? We could have a valid piece of mathematics rejected by journal referees, or conversely, we could have a flawed piece of mathematics accepted. Consider, for example, the 'proof' of the 4-colour theorem by Alfred Kempe in 1879 and Peter Tait in 1880: both 'proofs' were accepted as being correct until 1890 when it was revealed that they both contained fallacies (Katz 1993). That the fallacies existed was prior to the revelation that they existed! The editor of a prominent mathematics education journal, in collaboration of the respective referees, reacted to this argument by stating that [c *]ertainly a (to me) tenable socio-cultural view would be that it was true for about ten years around 1880 (that is accepted by the professionals as proved, though it is also arguable that sofew people cared that it didn't get a great deal of scrutiny) and then became contentious once again with the discovery of a flaw in the proof* Here we have truth as synonymous with the acceptance by the professionals, and yet within the same sentence we have the discovery of a flaw - a flaw which presumably existed prior to discovery! Ernest's (1998) shilly-shallying is exemplified in not knowing which way to turn - logical necessity or consensus:

> *Thus in mathematics, so long as a set of assumptions is agreed, as are the rules to be applied to them, the consequences are inevitable, and different persons will come to the same conclusions independently ....However agreement is achieved through consensus or victory in language games and forms of life rather than by reference to extramathematical absolutes* (p. 259).

On the one hand, *once* the assumptions are agreed then the consequences are inevitable and different persons will come to the same conclusions independently; yet on the other hand agreement is achieved *through* consensus or 'victory' in language games and forms of life. Surely, however, agreement *constitutes* a consensus rather than being *achieved through* consensus and is possible *because* the consequences are inevitable and can be realised by different people!

Is the certainty of mathematics a myth? The many educationalists who have argued that it *is* have based their argument on the 'fallibilist' philosophy of Ernest: if mathematics is 'mutable and fallible', as Ernest has attempted to show, then the 'absolutist' claim that mathematics is immutable and infallible (and hence certain) is therefore unwarranted. This is a mistake! Mathematics may be 'mutable' and 'fallible' as Ernest argues, *but that has no relevance to the certainty of, for example, Pythagoras' theorem.* Just because we cannot apply the criteria of certainty to the *foundations* of mathematics does not mean that we cannot be certain of many of its truths, and, in exactly the same way, just because we cannot reduce mathematics to logic (see Ernest 1991) does not mean to say that mathematics isn't logical.

The Pedagogical Consequences of Denying Mathematics as an Objective Body of Knowledge.

By downplaying mathematics as an objective body of knowledge, the relativism of social constructivism places the emphasis on the aspirations and experiences of the pupils whereby it is they that choose the issues and objects of study in accordance with their needs and perspectives. Ernest (1991) sees this as a *social construction of meaning stemming from the theory of the social origins of thought of Vygotsky* .... *This theory sees children as needing to actively engage with mathematics, posing as well as solving problems, discussing the mathematics embedded in their own lives and environments (ethnomathematics) as well as broader social contexts.* (p.208). This interpretation of Vygotsky (and Ernest and many constructivists ritually evoke Vygotsky) sees mathematics teaching based on everyday contexualised and situation-specific reasoning, and this view is echoed in Ernest (1998). One of the dangers of this view is the reduction of mathematics to a form of 'work experience' (for example, Masingila's *et al1996* proposal that the mathematics taught should be such that students can acquire the concepts and skills required to solve routine dilemmas in life, leaving you to wonder how working people coped in the first place). We as educationalists have to ask ourselves a very important question: what should determine the mathematics that is taught - the needs of industry and commerce (for the development of *labour power,* see Matthews 1980)1 or the needs of the pupil *to be encultured into an academic discipline?*

Despite all the allusions to Vygotsky, Vygotsky (1962, p.12?; 1994, p.351) speaks of mathematics as an 'ideal' and not something that should be context-specific or something to be socially or communally negotiated or something that can be developed simply with the child in social activity with others: *Imagine a child who will develop his concept of numbers, his arithmetical thinking, only among other children, who will be left to his own devices in an environment where no developed form of arithmetical thinking exists, rather than in school or in kindergarten, i.e. without any interaction with the ideal form of adults. What do you think, will these children get far in developing their arithmetical thinking? None of them will, not even the mathematically gifted ones among them. Their development will remain extremely limited and very narrow in scope* (Vygotsky 1994, p.351). The teacher can only facilitate an understanding of mathematics if the mathematics, as an ideal, is already present in the environment - namely the mathematics that is facilitated by the teacher. In other words: the teacher can only facilitate an understanding of mathematics if the teacher embodies the ideal • that ideal is mathematics as a body of knowledge.

1 Any reference to everyday, context -specific, and 'relevant' mathematics must, in the main, refer to mathematics in the world of work - there is very little mathematics practised in 'play' or at a 'rave' .

Borba, M. and Skovsmose, O.: 1997, 'The Ideology of Certainty in Mathematics Education', *For the Learning of Mathematics,* 17(3), Nov.

Chalmers, A.: 1982, *What is This Thing Called Science,* The Open University Press, Milton Keynes.

Ernest, P.: 1991, *The Philosophy of Mathematics Education,* The Palmer Press, London. Ernest, P.:

1998, *Social Constructivism as a Philosophy of Mathematics,* SUNY Press, Albany. Jaworski, B.:

1994, *Investigating Mathematics Teaching.* The Palmer Press, London.

Katz, V.: 1993, *A History of Mathematics,* Harper Collins, New York.

Masingila, J. O., Davidenko, S. and Prus-Wisniowska, E.: 1996, 'Mathematics Learning and practice in and out of School.' *Educational Studies in Mathematics,* 31, 175-200.

Matthews, M.: 1980, *The Marxist Theory of Schooling.* Harvester Press, Sussex.

Phillips, D. C.: 1998 'Coming to Terms With Radical Social Constructivisms', in M. Matthews (ed) *Constructivism in Science Education: A Philosophical Examination,* Kluwer, Dordrecht.

Rowlands, S. and Graham, E.: 1997, 'The Nature of Mathematics and the State of Mathematics Education', *Chreods,* no.1l, Spring. Manchester Metropolitan University and Nottingham University.

Sfard, A.: 1994, 'Mathematical Practices, Anomalies and Classroom Communication Problems', in P. Ernest (ed) *Constructing Mathematical Knowledge.* The Palmer Press, London.

Skemp, R.: 1976, 'Relational Understanding and Instrumental Understanding', *Mathematics Teaching,* 77, 20-26.

Taylor, P. C.: 1996, 'Mythmaking and Mythbreaking in the Mathematics Classroom', *Educational Studies in Mathematics,* 31, 151-173.

Therlfall, 1.: 1996, 'Absolutism or not Absolutism - What Difference Does it Make? *Philosophy of Mathematics Education Newsletter* no.9, 11/14/96 (http://www.ex.ac.uk/PErnest/pome/pompart6.htm) .

Vygotsky, L.: 1962, *Thought and Language.* M. I. T. Press, Cambridge, Massachusetts. Vygotsky, L.:

1994, 'The Problem of the Environment', in R. Van Der Veer and J. Valsiner (ed.) *The Vygotsky Reader*, Blackwell Publishers, Oxford.

Walkerdine, V.: 1994, 'Reasoning in a Post-Modern Age', in P. Ernest (ed) *Mathematics, Education and Philosophy.* The Palmer Press, London.

Wood, J. E.: 1980, *Sun, Moon and Standing Stones.* Oxford University Press, Oxford.