THE RESEARCH OF IDEAS OF PROBABILITY IN THE ELEMENTARY LEVEL OF EDUCATION

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Abstract Ideas of probability have been investigated in Mexican elementary education. Two examples are given to illustrate the way in which epistemological aspects are considered in this research. Teaching experiments with 6-7 year old children suggest that pupils' interpretations of the tasks they were asked about may result in answers which do not inform on their idea of chance, since they tend not to focus on it. Additionally, by using questionnaires and clinical interviews with 10-15 year old children, it was found that their correct handling of fractions does not assure that they can cope with questions about probability for which a quantification would suppose fraction use.

Introduction

Results from Piaget & Inhelder's studies about the origin of the idea of chance in children (1975) and from some researchers in mathematics education (e.g. Garfield and Ahlgren, 1988) have influenced importantly the curriculum of probability for elementary school. However, these views leave aside some aspects about children's interpretations of random situations, which seem to be important for the process of teaching. Weare concerned with children's idea of chance and with what their quantifying of probability may reveal.

The Idea of Chance in Young Children

Since ideas of probability are not included in the first two years of the Mexican primary school, Gurrola (1998) sought information about the convenience of introducing probability in these grades. Accordingly, six children of this school age were asked about ideas of probability within a teaching experiment protocol (Glasersfeld, 1983). When using this method, a clinical interview is prolonged until evidence is obtained from the interviewed child making clear his/her understanding of the situation he/she is asked about. Some results concerning the idea of chance in children aged 6-7 years are presented here.

A device was used to question children about random mixture. It consisted of a rectangular tray which can be made to swing up and down by means of a fulcrum fixed at its base. Twelve equally sized marbles, six white and six green, can be arranged on both sides of a divider that the tray has at the middle of one of its sides, before letting them free by balancing the tray (see

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Figure 1). According to Piaget & Inhelder (1975), children of the age considered here do not anticipate the irreversibility involved in random mixture; instead, they try to find any kind of order on the grounds of common properties of the elements or of their original arrangement. As a result, they are not able to start understanding what chance is.

Nevertheless, among the six we interviewed, four foresaw the mixture of the marbles when balancing the tray, that is, a different position from the one they had originally; the other two children proposed the marbles arriving at the same position they had at the start, but finished the teaching experiment stating that different positions should be expected. They realised that, as a consequence of the balancing of the tray, collisions between the marbles may well take place. Additionally, during the questioning, it arose that when interpreting the whole activity,

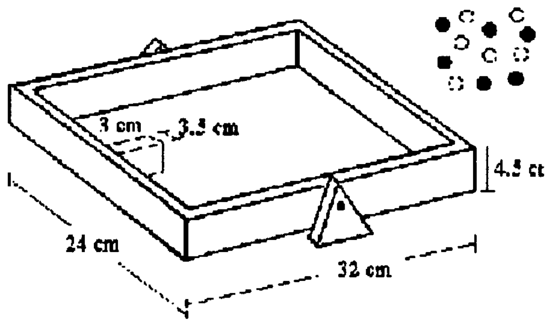

Figure 1

some children may not focus on chance, but rather on trying to overcome it. That is, they may interpret the situation as a task to control the movement of the tray so to obtain, after each swinging, the marbles arranged as originally. This was the case of Almendra (7 years, 9 months).

In the first arrangement, the twelve marbles were shown to the girl separated by colour at both sides of the divider (in the following transcripts, *E* stands for interviewer, *A* for Almendra):

E What do you think is going to happen if we balance the tray like this?
A One [marble] *can go here, another here and another here* [she describes lines towards the opposite side of the tray but in the same direction]. *But without crossing here* [as she points to the middle of the tray extending an imaginary line stemming from the divider]: *these are green and these are white.*

After one movement of the tray, the girl sees that some marbles have changed their position. Before each of the subsequent trials, she arranged the marbles as they' were presented to her at first. Apparently, she does not conceive the irreversibility of the mixture, but she states she does not know how to prevent the changing of the places between the marbles, and continues referring to the role of the divider in the device:

A This is meant to keep them [the marbles] *at their place.*

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Therefore, the divider was removed from the tray and the device was presented to her again showing the marbles mixed, that is, not obviously arranged by colour (p. 99):

E Do you think the greens will go back to their place and the whites to theirs?
A No.
E No? Why?
A Because the movement makes them change their places.

She even realises that the collisions between the marbles contribute to the mixture.

The teaching experiment cases examined in Gurrola's research suggest that, in fact, young children could be faced with the idea of chance by posing to them appropriate situations and questions, so they could focus on features of that idea, such as variation in the occurrence of results and the possibility of different arrangements. However, an inquiry in depth is required to know about the results of children's understanding of random mixture when this idea is introduced into the classroom environment by means of a didactical activity.

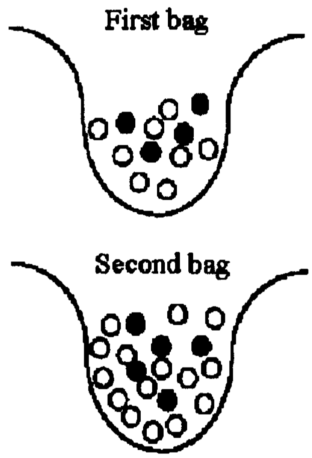
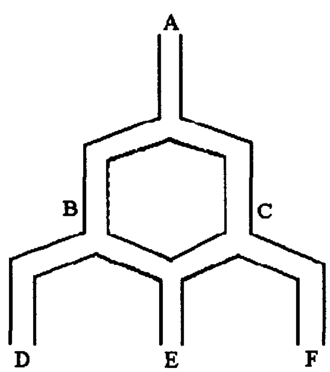
Arithmetical and Probabilistic Thinking

The other example we present here concerns what children's use of numbers may reveal about their understanding of probability, since some authors have pointed'to the deficiencies in handling fractions as one of the difficulties for that understanding (e.g. Garfield and Ahlgren, 1988). To analyse this point, Perrusquia (1998) aimed at obtaining information from 10-14 year old pupils (5^{th} and 6^{th} grades of primary school, 1^{st} and 2^{nd} grades of secondary school) when they assign fractional numbers to the likelihood of an event. After Kieren's work (1988), understanding rational numbers is shown when these numbers are used according to the different interpretations for which they are meant.

This qualitative oriented study had two phases. The first one consisted of a general inquiry involving 145 pupils who answered a questionnaire. The second phase was carried out with eight children chosen from the sample in the first phase.

The questionnaire used in the first phase was designed in an arithmetic version and in a probabilistic version, each of which included ten problems, two for each of the five basic interpretations of rational number (part-whole, measure, quotient, operator and ratio (1988». The questions posed in the probabilistic version referred to the same situations posed in the arithmetic version but now involving chance and to number assignation according to the answer given. These questions required the child to identify events, to express the probability of events from a partition of the sample space, independent events, and to decide which of two random situations offered higher probability for a given event. Examples of the questions posed are shown in Table 1.

Table I. **Examples of questions posed in the two versions of the questionnaire.**

Arithmetic version		Probability version
9. Some marbles are contained in two bags: in the first, there are twelve, eight are white and four are black. In the second bag, there are twenty marbles, fifteen are white and five are black. *What fraction of marbles in each bag are black?* ______ *Which bag has the biggest fraction of black marbles?* __________	(Ratio construct)  	*9. Some marbles are contained in two bags: in the first, there are twelve, eight are white and four are black. In the second bag, there are twenty marbles, fifteen are white and five are black.* *Without seeing, I want to draw a black marble. What bag should be chosen?* __________ *According to the bag you chose, what is the fraction of black marbles for that bag?* __________
10. Balls are let free in this maze. If one hundred and twenty are introduced at A, what fraction of the whole lot would come out through each of the exits D, E and F? *D* ______ *E* ______ *F* ______	(Operator construct)  	10. *Balls are let free in this maze. If one hundred and twenty are introduced at A, what exit, D, E or F do you think more balls will come out?* __________ *What fraction of the whole lot of balls do you think will come out through the exit you chose?* __________

Pupils answered the arithmetic version of the questionnaire a week before they dealt with the probabilistic version. The youngest pupils (27 children in the 5th grade of primary school) obtained the lowest rate of incorrect answers in the probability version (85%) of the ratio construct questions (the most abstract of the constructs we considered). For each of the school grades considered here, there was at least one pupil showing poor performance in fractions and a good performance in probability questions (for instance, Alberto, aged 10,6.2% and 70.5%, respectively).

For the second phase of this research, the highest rates of correct answers were taken as the reference to choose two pupils from each school grade: one having the highest score in the arithmetic version of the questionnaire (four in all), the other in the probabilistic version (four in all). These eight subjects were interviewed using a protocol of semi-structured questions based on their answers given to the probabilistic version.

According to the results, the children from the arithmetic group gave answers in the interview without referring to chance. Although their handling of fractions was correct, they showed a deterministic or linear approach to the questions posed. On the other hand, the pupils of the probability group accepted or even proposed the variation in the occurrence of possible events; they did not refer to a pattern to explain the probability they assigned to events. Only the oldest of these pupils (2nd secondary grade) considered equiprobability. As 'an instance, we present some transcripts for question 10 (in Table 1):

Question 10. Probability aroup (pp. 127-128)

E *Jfwe get 60 balls in A, which exit is more likely to have more balls come out?* G *Twenty or thirty?*
E *60.*
G *Then there would be 30* [pointing at D], *saying twenty* [pointing at F] *and 10* [pointing at E]; *or thirty here* [D], *twenty this side* [F] *and ten here* [E]. *It could vary a lot.*

Ouestion 10. Arithmetic group (p. 198)

E *Jfwe get 60 balls in A, what exit is more likely to have more balls coming out?* An *E.*
E *What part of the sixty* [will go out through E]?
An *Thirty over sixty* [she writes "E 30/60"].
E *What part of the sixty will be out through the less likely exits?*
An [She writes "D 15/60" and "F 15/60"].

The pupils of the probability group did not show a consistent performance in the arithmetic version. However, in the probability version, three of these four children answered correctly the questions requiring the ratio construct (for instance, question 9 in Table 1):

Ouestion 9. Probability group (pp. 172-173)

E *From what bag is it less likely to draw a black marble?*
D *Black ... from the second ... Cause ... in the first there are four and in the second five; then it increases but also there are fifteen whites in the second, and in the first there are eight; that difference of seven whites reduces chance.*

Question 9. Arithmetic group (pp. 200-201)

E *Let's consider two bags, the first having* 6 *whites and* 12 *blacks, and the second one white and* 5 *blacks. From what bag is it more likely to draw a black marble from?*
An *Thefirst ... Because there are more blacks.*
E ..... *If the first has* 5 *whites and* 3 *blacks, and the second three whites and* 1 *black?*

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An *The first one ... Because there are more whites.*

The 'probability' pupils interpreted the random situations from a frequential approach more often than the children from the arithmetic group, or quantified by means of percentages.

Remarks

The results obtained suggest the convenience of introducing didactical activities for teaching probability at elementary education by giving priority to a frequential focus as a natural approach pupils have to chance. Preferences and interpretations young children make of the tasks proposed when these involve chance should not be neglected as the aim pursued may not be attained. Finally, the fact that different basic interpretations of fractions can be required when facing questions about probability, offer an opportunity for the child to give sense to the use of these numbers by focusing on the study of chance.

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