DO KS3 SATs TEST FOR QUALITATIVELY DIFFERENT FORMS OF MATHEMATICAL UNDERSTANDING?

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Here I present case studies of individual pupils who, whilst achieving the same SA Ts levels, seem to display different forms of understanding of mathematics. This raises the question about whether KS3 SATs distinguish between procedural and conceptual understanding, as was originally intended.

Whether or not the KS3 SATs do actually differentiate between qualitatively different forms of understanding, is only important if they were actually intended to differentiate. The "Task Group on Assessment and Testing", responsible for the guidelines for the National Curriculum SATs, recommended that attainment targets:

> draw on a wide and complex range of subject based knowledge, skills and understanding.
> (DES, 1987, para. 158)

It is my interpretation that the SATs were designed to consider procedural and conceptual understanding separately. I propose to show that the KS3 SATs are not achieving this aim.

In 1997 the "School Curriculum and Assessment Authority" reported that:

> Most 14-year-olds are expected to achieve **Level 5 or** 6.
> (Original bold; SCAA, 1997)

The three pupils for whom case studies are presented, all took their KS3 SATs in 1997. Each achieved a Level Seven, that is, above the national expected level.

For the purpose of these case studies, three categories of understanding are considered: None Apparent; Procedural; Conceptual The first was applied when no understanding was identifiable from a pupil's response, for example the question was omitted. The difference between procedural and conceptual understanding coincides with 'knowing how', but not why, and 'knowing how and why' (Skemp, 1976, p. 23). A pupil with procedural understanding would be oblivious to the fact that an algorithm had been misapplied or incorrectly remembered, and their understanding would be non-transferable. It is not always possible to distinguish between an individual with procedural and conceptual understanding, as the latter could work procedurally, unless a question is designed for this purpose.

A question sheet was constructed to differentiate between these qualitatively different forms of understanding. Following detailed classroom observations, predictions of pupils understanding were made. These were then stringently tested against the question sheet in the pilot study to confinn that the questions did differentiate. The question paper was taken by the research group in the same week as the KS3 SATs. No additional topics were taught in between. Calculators were not used in my study.

The classifications used for the three pupils were determined by the form of understanding which they usually exhibited:

Pupil A: No Apparent Understanding;
Pupil B: Procedural Understanding;
Pupil C: Conceptual Understanding.

A selection of their responses to my question sheet will now be presented.

Questions in the SATs usually provide the algorithm required. I wanted to determine whether the pupil could select the appropriate model for this problem from multiple choice answers. This question was first used by Hart (1981):

On a motorway, my car can go 41.8 miles on each gallon of petrol.
Which calculation below shows how many miles I can expect to travel on 8·37 gallons?

8·37 ÷ 41·8	41·8 − 8·37	8·37 − 41·8
8·37 × 41·8	41·8 + 8·37	41·8 ÷ 8·37

Only pupil A failed to correctly select the required calculation, choosing division instead of multiplication.

The next questions to be considered is:

Express $^5/_{16}$ as a decimal.	Express $^5/_{16}$ as a percentage.
Answer = _ _ _ _ _ _	Answer = _ _ _ _ _ _

These questions were designed to determine whether the pupil had the conceptual understanding to appreciate that these two questions required the same calculations. The two questions were placed one question apart so that the relationship was not immediately apparent. A pupil who only needed to work out one solution, and could then derive the other answer, had understood the related concepts. Pupil A failed to attempt either of these questions, whilst pupil C performed the division correctly to give the decimal answer 0'3125, and went on to derive the second. Pupil B attempted to apply a rote learnt procedure to each, but both were either incorrectly remembered or applied. His solutions were:

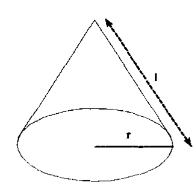
Decimal: $16 \overline{)1000}$ = $6{\cdot}\dot{2}\dot{1}$ Answer = 0·621	Percentage: $5 \overline{)16{\cdot}0}$ = 3·2 ×100 Answer = 32%

The following question was designed to determine firstly whether a pupil could apply an uncommon procedure, and secondly whether the limitations of the question were appreciated. Conceptual understanding was required for the latter: with the resultant dimensions, the shape could not possibly be a cone. This question is now copyright ofthe QCA.

The formula for the surface area of a cone is πrl where r is the radius and l the length of the slant.
Given that the surface area of a cone is $54cm^2$ and the radius is 9cm, find the length of the slant.
(Take π as 3).

l = _ _ _ _ _ _ _ cm

Comment on your answer.



The length of the slant was 2cm. Pupil A omitted both parts of the question. Pupil B attempted to calculate the answer, however, he muddled up the procedure, using subtraction instead of division as the inverse of multiplication. This highlights the disadvantage of procedural understanding, when the algorithm is not accurately remembered errors occur and are not detected. Pupil B's solution was:

Pupil B. $3 \times 9 = 27$

$$\begin{array}{r} 54 \\ -\,27 \\ \hline 27 \end{array}$$

$l = 27\text{cm}$

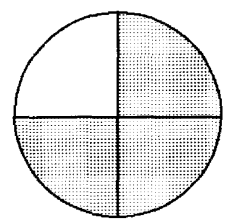
Pupil C correctly found the length of the slant to be two, and whilst he failed to appreciate that this could not be a cone, he attempted to apply conceptual understanding by stating that the answer was "a whole number, but would not be if 1t was used more accurately". Pupil B had just commented on the value of the slant obtained.

The next two questions were intentionally placed consecutively. The first had been used by Hart (1981) and I designed the second to complement it:

Shade in $^1/_6$ of the dotted section of the disc.
What fraction of the whole disc have you shaded?
Answer = _ _ _ _ _ _

AND

Calculate $^1/_6 \times {^3/_4}$.
Answer = _ _ _ _ _ _ _



The first question required the pupil to follow instructions and applied a visual means with which to do this. Procedural understanding was not required for this. Pupils B and C answered this question correctly, whilst pupil A failed to do this. The second question was designed to determine whether the pupil realised that the question was actually the same as the first, just given in a non-visual form. If a pupil appreciated this, they had demonstrated proceptual understanding, that is the symbol was represented ambiguously by both the concept and procedure (Gray and Tall, 1994). Pupil A again failed to attempt this question, whilst pupils Band C both incorrectly attempted to apply a rote learnt procedure. Both pupils made the same errors with the procedure, assuming that a common denominator was require and that only the numerators were multiplied:

Pupil B:

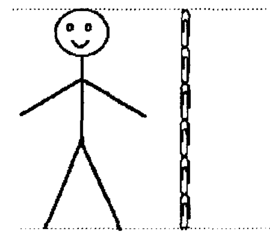
$$\underset{\times 2}{\frac{1}{6}} \times \underset{\times 3}{\frac{3}{4}} \quad \frac{2}{12} \times \frac{9}{12} = \frac{18}{12} = 1\,\frac{6}{12} = 1\,\frac{1}{2}$$

Pupil C

$$\frac{1}{6} \times \frac{3}{4} = \frac{2}{12} \times \frac{9}{12} = \frac{18}{12}$$

The next two questions to be considered were both ratio questions:

You can see the height of Mr. Short measured with paper clips.
Mr. Short has a friend Mr. Tall.
When we measure their heights with matchsticks:
Mr. Short's height is four matchsticks;
Mr. Tall's height is six matchsticks.
How many paperclips are needed for Mr. Tall's height? _ _ _ _ _
Explain your answer.



AND

In a particular shade of purple paint there is:

1 part yellow to 5 parts blue;
3 parts black to 10 parts blue;
8 parts red to 15 parts blue.

(a) You would need how many parts yellow to how many parts black?
Answer = _ _ _ _ _ _

(b) You would need how many parts red to how many parts black?
Answer = _ _ _ _ _ _

The first question was designed by Karplus (1975) and used a visual representation in order to assist pupils in obtaining the answer (cited Hart, 1981). The second question was a variation of one used by Hart (1981), where no assistance was given and there were more stages involved in the solution. It was therefore harder to apply a rote learnt procedure. The question had been amended to avoid the original context of metal alloys, with which the pupils were unfamiliar. Whilst the context of mixing paints was still an irreversible operation, it was one that all pupils were familiar with. The school possessed a very strong art department. All three pupils succeeded in answering the first question correctly. Pupil A, however, failed to attempt the second question. Pupil B provided two incorrect answers, neither of which were accompanied by workings. Only pupil C was able to answer both parts of this question successfully.

The final question to be considered was adapted from one of the 1992 KS3 SA Ts papers. The original question had required an answer that was unrelated to the context of the question. I had included this question on my paper, to demonstrate that pupils with conceptual understanding, in particular, would be disadvantaged by this, as they would appreciate the limitations of a numerical answer. It was for this reason that I added the extension: "Explain your answer". I was aware, however, that other pupils may benefit from this prompt. The question was:

There is one lift in an office block. One hundred and ninety-two people work above the ground floor. Given that the lift can carry eight people at one time, how many times will it be used in a morning?
Explain your answer.

Pupil A correctly performed the required division. However, he commented purely on the workings used to achieve the answer: "24 times as 8 goes into 192 24 times as shown in the sum". Pupil B also calculated that the answer to the division was 24, but gave as his :final answer: "24 (n", where tIn = number oftimes it will be used". Explaining that "some people may come in at different times , and so only 6 people may get into the lift at anyone time, leaving two people. These two people will have to use the lift an extra time to get to work". Therefore, whilst pupil B did not appreciate that the lift may be used less than twenty-four times, the limitations of the question were apparent to him. Pupil C, on the other hand, did not even bother to calculate the division. His answer was:

> You cannot tell. In the morning, not every one may be there. The lift will not just go when it is full, but when there are no more people to go in the lift. For example, the first person to use the lift may be so early that he is on his own, and he will not wait for the lift to fill before he goes. People may go down again, perhaps to get something from their car, or for lunch break. Other people, such as repair men, that do not usually work in the building may be needed and they would also have to use the lift. Some people may prefer to take the stairs.

These three pupils, who had been taught together since the age of eleven, displayed very different forms of understanding. All, however, achieved Level seven in the KS3 SATs, raising the question about whether they distinguish between procedural and conceptual understanding.

References

Department of Education and Science & The Welsh Office (1987). *National Curriculum: Task Group on Assessment and Testing. A Report.* England: H.M.S.G.

Gray, E., & Tall, D. (1994). Duality, Ambiguity, and Flexibility: A "Proceptual" View of simple Arithmetic. *Journalfor Research in Mathematics Education,* 25, 116-140.

Hart, KM. (1981). The Research ofCSMS. In KM. Hart, (Ed.), *Children's Understanding of Mathematics:* 11-16 (pp. 1-8). London, England: John Murray.

Schools Curriculum and Assessment Council (1997). *National school tests and teacher assessment in* 1997: *A leaflet for parents.* London, Great Britain; SCAA Publications.

Skemp, RR (1976). Relational Understanding and Instrumental Understanding. *Mathematics Teaching,* 77(4), 20-26.