DEVELOPING ALGEBRA - A CASE STUDY OF THE FIRST LESSONS FROM THE BEGINNING OF YEAR 7

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This is a case study from the start of a project, funded by the Teacher Training Agency (ITA), looking at one mixed ability year 7 *class in a Bristol comprehensive, investigating what algebra is taught and what algebra is learnt over the period of their first term at secondary school. The motivation for the study is to explore the challenge inherent in the quote: 'Can we develop a school algebra culture in which pupils find a need for algebraic symbolism to express and explore their mathematical ideas?' (Sutherland* 1991). *We see the role of the teacher as that of setting up a 'community of practice' (Lave and Wenger* 1991) *in the classroom, where the practice is that of 'inquiry' (Schoenfeld*, 1996). *Resultsfrom the first seven lessons with the group are presented and analysed, setting up further research questions.*

This paper is written in the fIrst person, from the perspective of Alf Coles, partly for ease of reading. However the research questions, lesson plans, reflections, analyses and the structure of the paper have all been jointly created with Laurinda Brown.

Introduction

I became interested in researching the teaching and learning of algebra after reading a recent national report into school algebra: 'Teaching and Learning Algebra pre-19' (Sutherland 1997). The report makes the following recommendations (amongst others):

> *'The National Curriculum is currently too unspecific and lacking in substance in relation to algebra. The algebra component needs to be expanded and elucidated - indeed rethought. '* (P29)
>
> *'We suggest that more research is needed to understand the relationship between what algebra is taught and what is learned.* ' (P30)
>
> *'We recommend that algebra is introduced from the beginning of secondary school with more emphasis being placed on all aspects of algebra.* ' (P30)

When the report calls for emphasis to be placed on 'all aspects of algebra' (P30) the reference is to Kieran's (1996) classifIcation of algebraic activity into three components. The three components are:

> *'(i) Generational activities - discovering algebraic expressions and equations;*
>
> *(ii) Transformational rule-based activities - manipulating and simplifying algebraic expressions, solving equations, studying equivalence and form;*
>
> *(iii) Global, meta-level activities - ideas of proof, mathematical structure, problem-solving. (This final component is not exclusive to algebra.) ,*
>
> (Sutherland 1997, p12)

Generational activities I take to mean activities where students investigate an ordered set of examples (eg how many matches do you need to make lxI, Ix2, lx3, etc rectangles?), which leads to a number sequence which can then be generalised (eg the number of matches needed for a Ixn rectangle). The algebra expresses the pattern in the numbers. Transfonnational activities come under the heading of classical algebra - at secondary school this entails manipulative techniques for simplifying expressions and solving equations.

Global meta-level activities include skills such as introducing a variable in order to solve a problem or to express the structure of a situation. In this category is the use of algebra as proof. 'Meta' implies the attempt to do something beyond the solving of the particular problem that is being facedtypically by establishing something about the general case or by expressing the particular problem in a more general manner.

Activity in this last category implies the ability to be involved in a process and also to be aware of what that process is. For example, expressing a process in an algebraic statement implies the ability to perform the process but also to view the process itself as an object. Tall's (1994) notion of a 'procept' captures this sense of being able to view a piece of mathematics as both process and object.

In a pilot study, I set a problem to two pairs of successful students, one pair in 10 and one pair in year 13, to see if Kieran's defmition was useful in practice. It was striking that the older students worked numerically on the problem until they gained some insight into its structure (a 'meta' awareness) and then effectively introduced symbols and arrived at a solution, showing evidence of all three components of algebraic activity. The year 10 students reached for symbolism more quickly but then became bogged down in transfromational work. This case study led me to question whether it is possible to work with a year 7 class so that the 'meta' aspects of algebra come fIrst, so that they see the power of algebra and get some sense of why they might need it? These questions are encapsulated in the following quote, which the current case study is designed to address:

> *'Can we develop a school algebra culture in which pupils find a need for algebraic symbolism to express and explore their mathematical ideas?* ' (Sutherland 1991)

For the purposes of this study I have adopted Kieran's (1996) defmition of algebra.

The first seven lessons

I have picked four sequences from the fIrst seven lessons with the year 7 class on whom the research is based, which give a sense of the classroom ethos that I am attempting to establish in order to achieve the aims set out in the introduction. I will give an *'account of'* (Mason 1994) those sequences based on lesson observation notes (by Laurinda Brown), my own research diary (written up soon after each class) and the students' exercise books. I will analyse each sequence in turn, to explain how it links with the aims ofthe case study. The evidence is indented.

Sequence 1: The beginning of the year

> I began by articulating the purpose of the year for the students as being about becoming a mathematician and thinking mathematically ie: thinking for yourself and so not asking me if things are right, noticing what you are doing eg patterns but then asking why patterns work, writing down everything you notice, being organised, doing things in your head. (AC research diary 4/9/98)

I wanted to establish a 'purpose' (Brown and Coles 1996, 1997) for the year that would be removed from the content level of what we did in class and would be an easily stated label that could accrue complexity and meaning for each individual as the year progressed. I hoped the purpose of 'becoming a mathematician' would support students in becoming aware of what they did when working in a mathematics lesson, by allowing them (and me) to question and reflect on whether something they (and I) did was mathematical or not.

Sequence 2: The ftrst activity

> I issued the following instructions, at the same time going through an example on the board:
>
> Pick any three digit number with 1 sl digit bigger than 3rd
>
> Reverse the number and subtract
>
> Reverse the answer and add

$$\begin{array}{r} 752 \\ -\ \underline{257} \\ 495 \\ +\ \underline{594} \\ 1089 \end{array}$$

> Several comments were made by students that they also got 1089 and the challenge I gave to the class was: 'Can you fmd a number that does *not* end up as 10891' (AC research diary 4/9/98)

There were several reasons behind the choice of this activity as the ftrst lesson with the group. One is that it is an activity I am familiar with but more important for my purposes was the fact that it is both self-generative and self-checking. By that I mean that, having set up the task I do not need to direct students as to what numbers to try out, they can generate their own examples to try. Also, some students typically become convinced very soon that all answers will end up at 1089 (which in fact they do), so if a student gets a different answer I can get them to check their answer with someone who thinks that is impossible, so the task becomes self-checking amongst the students. Both these elements leave my attention free to eg notice aspects of mathematical thinking that I can highlight to the group and also gives the students an immediate experience of having to 'think for themselves' which I had said was a part of 'becoming a mathematician'.

Sequence 3: Establishing ethos

One ofthe tasks I set myself for the year was to 'meta-comment' Bateson (1972) as much as possible on activity that I considered to be mathematical. Below are two examples, from the ftrst lesson:

> 'This group noticed something about their answers - it proved not to be 100% correct but it's an example of what it means to think as a mathematician.'
>
> 'This group had an idea which they wrote down and tested and found it didn't work so they changed their idea, that's a great example of what it is to think mathematically' (AC research diary, 4/9/98).

Meta-commenting in this way is not from a belief that my behaviours are transferable in any way but more from an attempt to set up a 'community of practice' (Lave and Wenger, 1991) in a classroom in which students are 'apprentices' in the act of inquiry into mathematics. Such a community:

> *'offers exemplars (which are grounds and motivations for learning activity), including masters, finished products, and more advanced apprentices in the process of becoming full practitioners* "' *it crucially involves participation as a way of learning - of both absorbing and being absorbed in - the "culture of practice".* , (P95)

A 'community of inquiry' (Schoenfeld, 1996) is such that *'the real 'authority' is not the Professor it's a communally accepted standard for the quality of explanations and our sense of what's right'* (p.16). The self-checking nature of the ftrst activity supported the development of this sense of inquiry. In meta-commenting and asking questions (eg 'The question I always ask is - why is this happening?') I am acting as a role model in this community.

Sequence 4: Algebraic proof

The following sequence took place in the sixth lesson on this activity, during which time different students had worked on different questions, some mine, some their own:

> With twenty minutes to go I stopped everyone and went through the work shown here, which I introduced as a way of proving what we found out for three digit numbers. Alongside the algebra I did a numerical example and at each stage of both the numerical and algebraic example I elicited answers for what to write from the class. I then wiped the proof off the board and set the class the challenge of reproducing i and then extending it to t prove things they had found out about the problem with different numbers of digits. (From LB lesson notes 21/9/98)

$$\begin{array}{cccc} & \cancel{a}^{-1} & \cancel{b}^{-1+10} & c^{+10} \\ - & c & b & a \\ \hline & a-1-c & 9 & c+10-a \\ + & c+10-a & 9 & a-1-c \\ \hline 1 & 0 & 8 & 9 \\ \hline & {}_{1} & & \end{array}$$

I only attempted this proofwith the class because we had worked on the problem (looking at three, four and ftve digit numbers) for so long that the students seemed comfortable enough with the process to be able to look at that process as an object. The algebra could be said to hold that process.

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I was keen that their first experience of algebra in secondary school would be one that was at this 'meta-level' and in a context where it allowed them to do something (in this case proof) that they would not be able to do without it.

Results so far

The aims of the case study were to do with whether it is possible to get year 7 students working on meta~level algebraic activities as a way of creating a culture in which algebra is meaningful and had a purpose. I have described mechanisms I used for this end, the results must come from an analysis of what types of activity students are actually participating in. In particular, is there evidence of meta-level activity?

At the end of the seventh lesson I asked the class to write anything they could under the heading 'What have I learnt?' since arriving at secondary school. I asked them to think about what bits of mathematics they had learnt and also what they had learnt about what it is to think mathematically. A majority ofthe class were able to write about something in both categories, for example:

> 'I've learnt that you have to think about the problem and not just do the sum. Also you have to maybe carry on thinking about the problem and see if it carries on. You could also have suggestions on why there are problems and how the problem works. It is a lot different to primary school because at primary school we just had to do the sum, we didn't think about the problem of the sum we had to just do it.'

I take this as evidence that the student sees part of their task in lessons as trying to become aware of what they are doing at the same time as doing it 'you have to think about the problem and not just do the sum'. This, to me, is the essence of meta-level activity. Another student puts it differently:

> 'I've learnt mathematicians have to think quickly and solve a lot of problems. You've got to jot little theories down. On a lot of theories you have to write why. You've got to correct your mistakes. You've got to confirm things ... You got to explain your fmdings.'

But again what the student is mentioning are meta-level activities - creating theories, proof. Another student picks up on aspects of mathematical thought that I was not aware of having stressed:

> 'I have learnt its OK to make mistakes ... maths is more exciting in secondary school than in primary because in primary school we copied from texts book and that it so boring I hated maths but I like it here because you can write on the board and make suggestions and talk about the work and write in our books'

I take this as evidence that students are creating their own meaning in relation to the notion of 'becoming a mathematician'.

Conclusion - further research questions

While looking at a selection of students' books from the whole of year 7, the head of department at my school commented that he had never seen students writing as much as in the class this study is about. The sense of 'becoming a mathematician' and the stress I have put on writing is not part of the school culture - however there is evidence of students across the range of achievements beginning to work in a 'global meta-level' way. For some this is at level of an awareness of having learnt how to subtract, for others it is in the ability to use letters to express their ideas and be comfortable with notions of algebraic proof. Mechanisms that have supported this seem to be my own 'meta-commentary' on the events of each lesson, the emphasis on students' written and verbal communication and the self-generative, self-checking nature of the activities in lessons.

Questions that the fIrst seven lessons have raised for me are: will other students in the class begin to take on board the use of algebra to express processes and ideas? will students begin to use algebra in new situations as a way of making sense and meaning for themselves? will the meta-level control of some students lead to effective ways of working on the generational and transformational aspects of algebra? I hope to report on some ofthese questions in future BSRLM meetings.

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