Cabri as a cognitive tool - Part 1

Catia Mogetta Graduate
School of Education
University of Bristol

A cognitive tool?

The main aim of this paper is to analyse the role of Cabri-Geometre[1] (Baulac, Bellemain & Laborde, 1988) as a cognitive tool in the teaching and learning of mathematics.

In the recent literature on the role of ICT in education computers are seen as tools to develop an original and independent thinking:

> Computers are cognitive tools 'as they serve to aid students in their own constructive thinking, allowing them to transcend their cognitive limitations and engage in cognitive operations they would not have been capable of otherwise' (Pea, 1987).

Following this approach to the use of technology in the mathematics classroom, Cabri is seen as a support for learners to transcend cognitive limitations and to construct a new relation to knowledge.

This paper gives an outline of one of two separate but related PhD projects, currently carried out at the University of Bristol, taking two different perspectives to the same central issue: the first project focuses on the role played by the student as learner/user, while the second one looks at the teacher as user and therefore producer of knowledge through the software environment.

Proof as a process.

The first PhD project is centred on the issue of the teaching and learning of mathematical proof and aims to analyse the role of Cabri in the process of proving statements.

Mathematical proof is a particular type of discourse, which has different functions in different settings: school mathematics has peculiarities related both to the aims and to the content which has to be taught. In this particular context a proof needs to be convincing and explaining (Hanna & Jahnke, 1993): the convincing function being related to the syntactic structure of the mathematical discourse and the explaining one being related to the content of the propositions, which acquires a particular relevance from a motivational point of view.

Taking a more general perspective to the issue, it is worth highlighting the fact that proof has specific aims in the construction of mathematical knowledge. The introduction of the idea of validation, as a necessary aspect of the mathematical activity needs to make use of proofs of

1 The proposed study intends to use the second version of the software, Cabri II, but for the general description of the functions and role it plays in the process of teaching and learning it is not essential to choose one of the two versions.

statements or, at least, rigorous justifications. This basic idea supports a collective and long term process of construction of a theory, whereby theorems are seen and introduced as a cognitive unity of three elements: statement, proof and theory (Mariotti et al. 1997).

In my work, a proof is not seen just as the final product of a reasoning conducted according to specific rules, but as the whole process of conjecturing, justifying and formalising solutions to mathematical problems.

Arguing vs. proving?

The elaboration of a mathematical proof involves two different types of reasoning, presenting conflicts and similarities at the same time: arguing and proving.

The use of an argumentation is strictly related to the semantic content of the propositions involved and to the perceived truth of the statements, mainly based on the plausibility of the arguments produced to support the justification. These arguments often draw on empirical and intuitive observations, which mislead the learners, highlighting not relevant aspects of the problem situation they are analysing; at the same time the structure of the argumentation is such that the deductive steps do not always lead to a unique conclusion and sometimes add information to the initial statement.

On the other hand, a proof involves just the syntactic aspects of the propositions and aims to provide validity, based on mathematical reasons and on the acceptability of the arguments on the basis of the conventions established by the mathematical community. Formal and rigorous arguments need to be provided in order to substantiate this validity and the deductive steps follow wellestablished rules, which lead to statements not adding any information to the initial one.

Between informal and formal reasoning in geometry.

The focus of this paper is on geometry as the ground where mathematical proof can be taught in an effective way, given the deductive structure of the subject matter. Students are likely to face some difficulties in solving geometrical problems, due to some intrinsic epistemological obstacles (Brousseau, 1986).

Visualisation processes fulfil specific epistemological functions, such as the identification of configurations, the illustration of a statement by space representation and the heuristic exploration of a complex situation. The choice of a particular configuration for the problem being tackled is often based on perceptions and empirical arguments and reflects only one of the possible ways of representing the problem statement. Another obstacle intrinsic in this sort of processes is the translation from the verbal to the graphical register, which requires a conscious use of well conceptualised mathematical objects and facts.

Other epistemological obstacles are strictly connected to representation processes: the choice of a representation for the problem situation involves the use of different registers of the mathematical discourse and therefore a flexibility in connecting different aspects of the same concept.

The pictorial representation which is prevailing in the solution of geometrical problems can create some difficulties and, in a sense, misconceptions, due to the intuitive evidence of the statements. In the case of geometrical figures, the evidence of the properties is particularly highlighted and can heavily influence the process of conceptualisation, and at the same time , the identification of properties and relations linking the different elements constituting the figure.

Learners have to face difficulties of a different nature in both visualisation and representation processes and the intermingled presence of informal and formal arguments supporting these processes needs to be acknowledged explicitly in order to enable the students to overcome the obstacles.

Bridging the gap: how does Cabri help?

It is generally accepted that there is a cognitive and structural gap between the argumentative and the proving practice: what I would argue is that it is possible to bridge the gap provided that suitable learning environments are designed with respect to the needs of the learners to get to know the structure of the mathematical discourse and its relation to a wider mathematical theory whereby it has to be carried out.

Cabri-Geometre is a microworld (Hoyles, 1991), which can provide the needed scaffolding to construct mathematical arguments and link them in a rigorous way, starting from the visual perception of the figure and an exploration of its characteristics and properties.

Some of the features of this software can support the introduction of the idea of mathematical proof

The dynamic representation of a problem statement helps the learner to visualise the objects in a particular configuration and then modify it in order to span the range of all the possible configurations, keeping the invariance of the relations between the objects.

The use of commands and functions in order to construct and then explore a figure requires the implicit reference to the underlying theoretical structure. I would argue that the correspondence between Cabri commands and the axioms and theorems of Euclidean geometry can help the students to substantiate their arguments with mathematically based reasons and therefore support a process of step by step construction of the solution and of the related justification/proof

The features of Cabri as a micro world can be important to provide a conceptual feedback to the learner: after playing a construction and exploring the properties of the figure represented, the dragging function, under which the inherent properties are kept invariant, and the property checker are useful means to understand the nature of the relations and the operations involved in the solution process.

While in a traditional paper and pencil environment the provided feedback is merely of a visual nature, Cabri offers a way of testing the mathematical features of the problem under consideration and, at the same time, visually demonstrates the holding properties.

Given these considerations, the main hypothesis of my study can be expressed as follows:

HYPOTHESIS - The co-existence of formal and informal aspects in problem solving activities performed within Cabri, and the necessity to follow well determined rules can minimise the effects of the pre-existing rationality on the development of a correct meaning of mathematical proof

How to implement suitable problem solving activities?

The main problems lies in finding a suitable and effective implementation of problem solving activities meeting the aims pursued. On the basis of my previous studies (Mogetta, 1996) the role of the characteristics of problems seems to be relevant: in order to provide a fertile ground for the evolution of a sense of justification in mathematics, problems should be open-ended, requiring a conjecturing phase and a consequent exploration of the set situation and not intuitively evident.

The main issue is the construction of a link between mathematical properties and explanations provided in order to justify their validity. The affordances of Cabri seem to provide the opportunity to build up this link and to reflect on the nature of the geometrical objects and their manipulation.

I'm currently designing a pilot study, with the main purposes of analysing the processes involved in the transition from the conjecture to the formalisation of the solution. The underlying idea is that Cabri fosters both the exploration of the properties and relations characterising the problem situation and the justification of the observed properties. The focus is on this particular phase of the whole process and it seems to be crucial in terms of the construction of a mathematical sense of justification, preliminary to the introduction of mathematical proof as a discourse (but not only).

From Informal Proceedings 18-2 (BSRLM) available at bsrlm.org.uk © the author

References

Brousseau, G. (1986), Fondements et methodes de la didactique des mathematques., *Recherches en didactique des mathematiques*, 7, 2, pp. 33-115

Baulac, Y., Bellemain, F. & Laborde, J. M. (1988) *Cabri-geometre, un logiciel d'aide a l 'enseignement de la geometrie*, logiciel et manuel d'utilisation, Paris: Cedic-Nathan.

Duval, R. (1992-93), Argumenter, demontrer, expliquer: continuite ou rupture cognitive?, *Petit x n03I,* pp. 37-61. Duval, R. (1998), Geometry from a cognitive point of view, in: Mammana C., V. Villani V. (eds) *Perspectives on the Teaching of Geometry for the 21st Century* Dordrecht: Kluwer pp. 30 - 43.

Hanna, G.- Jahnke, H. N. (1993), Proof and Application, *Educational Studies in Mathematics, v.* 24, pp.421-438. Hoyles, C. (1991), Developing Mathematical Knowledge Through Microworlds, in A. J. Bishop et al. (Eds.) *Mathematical Knowledge: Its Growth Through Teaching,* Kluwer Academic Publishers, pp. 147-172 Laborde, C. (1993), The Computer as Part of the Learning Environment: The Case of Geometry, In: C. Keitel and K. Ruthven (Eds.) *Learningfrom Computers: Mathematics Education and Technology*, NATO ASI Series, F 121, Springer Verlag, Berlin, pp. 48-67

Mariotti et al. (1997), Approaching geometry theorems in contexts: from history and epistemology to cognition, *Proceedings of the 21th PME Conference,* Lahti, pp. 1-180-195.

Mogetta, C. (1996), *Argomentare e dimostrare in situazione di problem solving*, non-published dissertation, Mathematics Department, University of Pisa.

Pea, R. (1987) , 'Cognitive Technologies for Mathematics Education', In: A. H. Schoenfeld (Ed.) *Cognitive Science and Mathematics Education,* Hillsdale, NJ: Lawrence Erlbaum, pp. 89-122