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MENTAL CALCULATION STRATEGIES EMPLOYED BY TEACHER-TRAINEE STUDENTS

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First year student teachers were asked to mentally solve different types of addition and subtraction problems typically found in mental arithmetic practice tests. Subsequently they were asked to describe how they found the answers, in terms of figuring skills such as counting-on, counting-back, counting-up, using number bonds etc. The student replies were analysed and classified according to which figuring skills were applied to which type of problem. Summaries of these figuring skills applied to the decontextualised question numbers were also demonstrated diagrammatically.

Background

It is generally recognised that the written signs of arithmetic do not have a one-to-one correspondence with the mental counting strategies used to solve numerical problems. For example, the mental skill of *counting-back* is not the same as *counting-up,* but the computation' 11 - 6' can be carried out either by counting-back 6 from 11 OR by counting-up from 6 to 11 (see for example, Womack, 1998). In a previous paper (Womack & Williams, 1998), an account was given of how these different counting skills were made overt, orextemalised by having 5 year-old children walk up and down on stepping stones. In that scenario, children invented signs for the different counting strategies to be undertaken. The same signs were then used to instruct children's mental strategies along an intemalised model of those stepping stones. The rationale for that investigation was based on the evidence from many quarters that children's (and adults') understanding of numbers is based on an intuitive 'theory of numbers' in which ordered number symbols represent *positions* in a hierarchical sequence, rather than *sizes* of individual sets (e.g. Dehaene,I997; Chatley, 1983). The 'popularity' of this model can be verified by personal introspection and classroom investigationirrespective of whether children have participated in stepping stone like experiences.

Aim

The primary aim was to determine to what extent these same mental strategies were used throughout school and into adult life. Further aims were to determine whether students could identifY their mental method, and also whether substantial proportions of students used similar methods for the same kind of problem.

Procedure

In this informal survey, seventy students were given five 'sums' in their traditional format or as simple standard word problems, and asked to 'compute' the answer mentally. They were then asked to describe the strategy they used. Each student was then given a question sheet (Figure 1), on which to record how they had carried out the calculation.

7

Prior to distribution of the question sheet, a brief explanation of the difference between counting-onlback and counting-up/down was given, but whether or not this counting tenninology was understood, students were nevertheless asked to make clear their method.

Figure 1

QUESTION SHEET	
How did you find the answers to the following questions?	
Question 1:	31 - 3
Question 2:	27 + 4
Question 3:	31 - 28
Question 4:	It is 28th March. Your birthday is 31st March. How many days to wait?
Question 5:	It is 3rd December. How many days until 31st December?

Results

Students' responses included the following methods. *counting-onlback. E.g. 25 count-on 3 is: 26, 27, 28, *counting-up/down E.g. 24 count-up (to) 29 is: 25, 26, 27, 28, 29, = 5, *jump-onlback & jump-up/down (i.e. 'chunking' the counting), *written algorithm methods - performed mentally,
*number fact knowledge.

These strategies were augmented by compensation methods (e.g. adding-on too much and taking-off later to compensate), and decade adjustment (using the proximity of a decade point to assist in the calculation). These methods are shown here by arrow and trapezium diagrams (Figure 2). (The diagrams are for the purposes of this paper and were not used with the students.)

Responses were analysed and classified according to the above categories, and the number and percentage of students using each method is given in Figure 3.

Summary of responses

A brief summary of methods used follows:

Question 1: 31- 3

Most students recognised this as a 'make less' problem and that the decade point 30 is immediately less than 31. Therefore studentsjUInped-back (1) to 30 and counted-back the remaining 2 numbers.

Question 2: 27 + 4

Most students recognised this as a 'make more' problem and that the decade point 30 is immediately more than 27. Therefore students jumped-up to 30 and counted-on the remaining number (1).

Figure 2

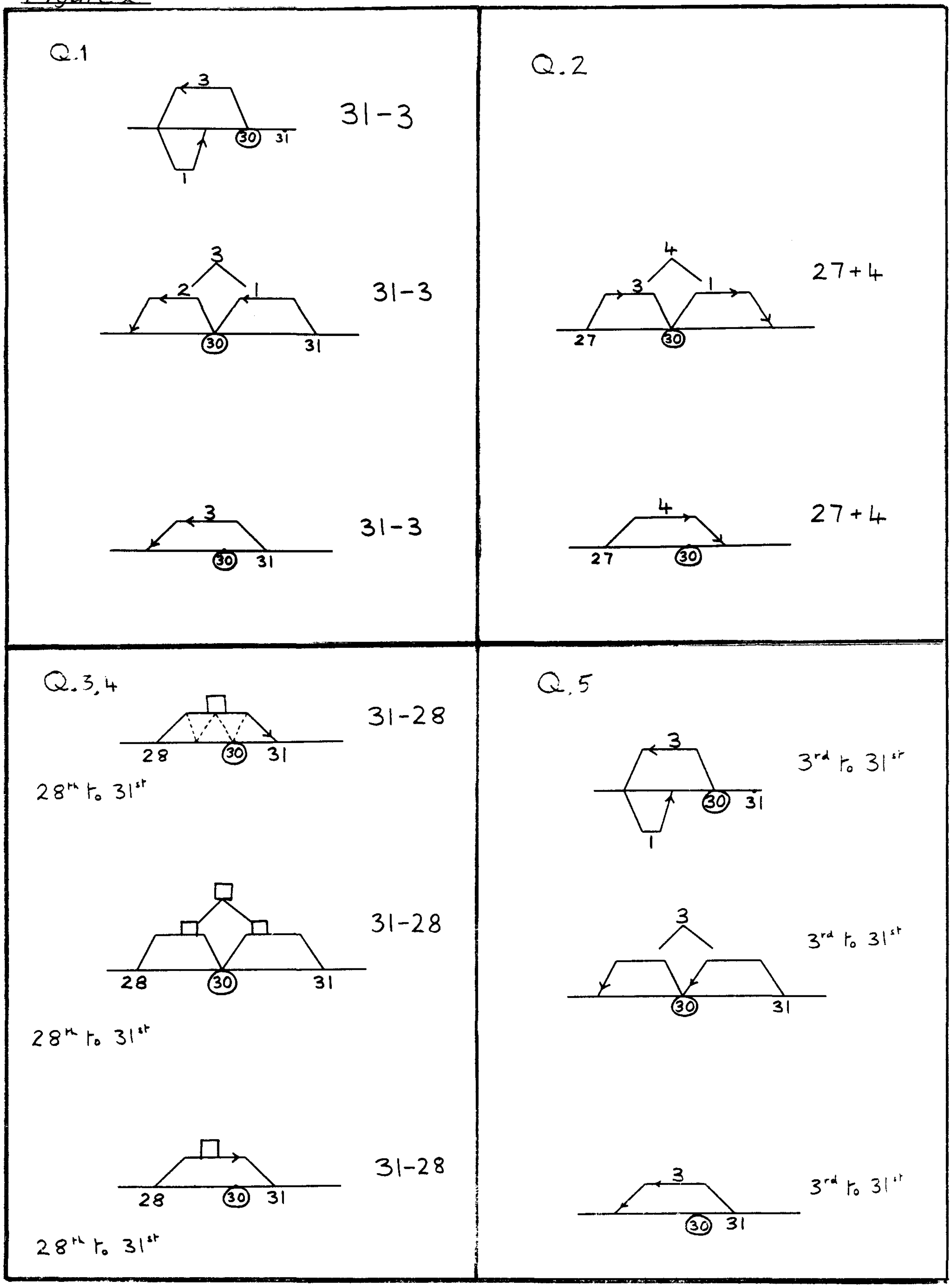
Q.1
31-3
31-3
31-3
Q.2
27+4
27+4
Q.3,4
31-28
28th to 31st
31-28
28th to 31st
31-28
28th to 31st
Q.5
3rd to 31st
3rd to 31st
3rd to 31st

Figure 3

Number & Percentage of students using each mental method

Question 1: 31 - 3

Methods:

	Number	%
1. Count-back	32	46
2. Count-back & decade adjustment	17	24
3. Count-back & compensation	12	17
4. Other	9	13

Question 2: 27 + 4

Methods:

1. Count-on & decade adjustment	26	37
2. Count-on	20	29
3. Written algorithm (mentally)	12	17
4. Other	12	17

Question 3: 31 - 28

Methods:

1. Count-up	28	40
2. Jump-up	17	24
3. Jump-up & decade adjustment	5	7
4. Written algorithm (mentally)	5	7
5. Various comp. methods	8	11
6. Other	7	10

Question 4: ***It is 28th March. Your birthday is 31st March. How many days to wait?***

Methods:

1. Count-up	51	73
2. Jump-up	5	7
3. Jump-up & decade adjustment	2	3
4. Jump-down & decade adjustment	2	3
5. Other	10	14

Question 5: ***It is 3rd December. How many days to 31st December?***

Methods:

1. Count-back	15	21
2. Count-back, decade adjustment & compensation	16	23
3. Count-up	3	4
4. Number-fact knowledge	10	14
5. Other	26	37

Question 3: 31 - 28

Most students recognised this as a 'find the numbers between' problem. Therefore students counted-up from 28 to 31 (in ones or a simple jump). ie three numbers altogether.

Question 4: *It is 28th March. You, birthday is 31st March. How many days to wait?*

Most students recognised this as a counting-forward problem from 28 to 31. That is, they used the same strategy as in Question 3.

Question 5: *It is 3rd December. How many days to 31st December?*

Most students recognised this as a 'make less' problem and counted-back three numbers to reach 28. That is, although the question was explicitly concerned with the ordinal positions of numbers, counting-up was not used as a strategy. It was the relative sizes of the numbers which determined the solution strategy.

Conclusion

It seems it may be possible to instruct students in appropriate mental strategies for certain kinds of problems - provided that students have not already developed a personal computational strategy which they use confidently and efficiently. This is a similar policy to that adopted when teaching written algorithms.

It may also be possible to utilize the arrow/ trapezium diagrams to *instruct* students to employ a certain mental strategy. Such an approach is widely used in overseas primary mathematics schemes such as the *Primary Mathematics Project Team* of Singapore and certain Hungarian schemes. Such a preliminary pedagogy would release pupils from the problem of deciding which mental strategy to use and allow them freedom to practise the different skills involved in the various counting strategies.

Finally, on the basis of this sample of seventy students, it appears that students' solution strategies are determined not solely by the *semantic structure* of the problem, but by the *relative sizes* of the numbers involved and the *position of the decade points* adjacent to the numbers.

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