ARE MENTAL CALCULATION STRATEGIES REALLY STRATEGIES?

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Some of the approaches used by children when calculating involve a flexible and inferential use of number knowledge. These approaches are often called "strategies", and it is a reasonable ambition for teaching to enhance children's skill in this area. But what exactly does effective performance of this kind involve? This paper considers to what extent children's calculation 'strategies' are really strategic, and explores the viability of teaching mental calculation through direct instruction in "strategies".

The sort of thing with which I am concerned are the methods by which calculations are tackled mentally other than counting or making a little visualised sum in the mind. Here are some examples from 10 year old children in a local school. They were given as a response to the question "How did you work it out?" following the requirement to calculate. Each set was taken from the independent responses of just 12 children of varying abilities (a different twelve in each case).

374 + 58

"374 add 6 is 380, add 2 is 382, add 50 is 432"
"I added 370 and 50 then 8 and 4 then added them all together."
"Make 374 up to 380 by adding on 6. That means you have 2 units left from 58. Then add the 50 onto 380 to make 430, then add your 2 units on to make 432"
"74 and 58 are 132, add 300 is 432."

1000 - 23

"1000 take away 1 is 999.999 take away 3 is 996. 996 take away 20 is 976, add one makes 977." " 1000 take away 20 makes 980. 980 minus 3 is 977."
"1000 is 970 and 30.23 off 30 is 7, so it is 970 and 7."

8 X 7

"4 times 8 is 32.3 times 8 is 24.24 add 32 is 56."
"8 times 1 is 8, 2 times 8 is 16,5 times 8 is 40,6 times 8 is 48, then 7 times 8 is 56, so it is 56."
"First I worked out what 5 eights were then I added 16 to get 56."
"First I times 4 by 7 and that is 28 then double the answer and you get 56."

1 02 divided by 4

"I did 4s into 20 goes 5 and there are five 20s in 100 so 5 times 5 is 25 and there is 2 left over."
"100 divided by 4 is 25, then divide 2 by 4 to get a half, so the answer is 25 and a half."
"I did 12 times 4 is 48 then 48 times 2 is 96 so that would be 24 fours, then another 4 is 1 00 with 2 remaining. "

It seems that whatever the question, children will find different ways to approach the problem. At least three different ways from only twelve cases in each instance, a level of variability that was by no means confined to these examples, as can readily be verified in any class of children (unless rigorously 'trained' to respond in particular ways). Given the same or a similar challenge, most adults report what they do in similar terms, suggesting that the "strategies" of children and adults are essentially the same.

Approaches to mental calculation have been sorted and labelled in different ways. Thompson (1997a and 1997b) for example, refers to:

- Derived fact approaches, where something that is known is altered to find an unknown
  e.g. 46 + 57 as 50 + 50 (=100) + 7 - 4 (=103)
- Cumulative sums: Breaking one number into parts and adding it bit by bit to the other number.
  e.g. 46 + 57 as 46 + 50 (=96) + 4 (=100) + 3 (=103)
  (This is called a "number-ten" method by Askew 1998 and others)
- Bridging-up-through-ten methods, where the cumulative sum is targeted on a ten or multiple of ten, e.g. 46 + 57 as 57 + 3 (=60) + 43 (=103)
- Partial sums: Breaking both numbers into parts and adding sets of parts together, usually the tens and the units etc., before re-assembling the whole.
  e.g. 46 + 57 as 40 +50 (=90) and 6 + 7 (=13) then 90 + 13 = 103
  (A "ten-ten method" in Askew 1998)
- Cumulo-partial sums: A hybrid, involving breaking both numbers up into parts, adding one set of parts then adding the remaining pieces progressively.
  e.g. 46 + 57 as 40 + 50 (=90) then 90 + 6 (=96) + 4 (=100) + 3 (=103)

It seems to be proposed, in the National Numeracy Strategy and elsewhere, that methods like this be taught directly and explicitly, under the umbrella name of "strategies". (The names used in the Numeracy Strategy are different, perhaps to protect the innocent.) Most descriptions of such teaching involve showing and practising each approach on suitable examples. Some additionally involve discussing with children how to choose an appropriate strategy for a given calculation (e.g. Clarke 1997).

While agreeing wholeheartedly with the idea that children should be taught to be more efficient and effective in mental calculation (to borrow a phrase from Askew 1998) I am concerned that explicit instruction in methods will not in fact achieve this. I believe that the implicit assumptions behind the approach, which in essence are that each method can be learned and applied as a strategy, are misplaced. Of course in a controlled classroom situation one can make progress in training children to respond in particular ways to carefully selected examples, and if sufficiently well flagged, children can come reliably to use a "derived fact" approach based on a known double to answer such questions as 48 + 53, for example. The question is what they are supposed to learn from this which is of relevance to a calculation demand which is not framed by the teacher or the context.

In everyday language the word "strategy" is used to mean two related things:

(i) a decision to do something in a particular way

(ii) a series of actions (in this case mental processes) by which a prior decision is carried out

The assumption behind the teaching approach seems to be that, having acquired a set of "strategies" through instruction, these are applied in actual calculation as the result of strategic decisions about which calculations methods are appropriate to a given challenge.

In the structured teaching situation the decision about what to do is made for the child by the teacher, in effect, through the teacher's intention to practice particular approaches. In the context of more open situations, where there is unanticipated calculation demand, the decision about which "strategy" is appropriate must be made by the children themselves, presumably on the basis of a kind of analysis of the problem. In the Numeracy Strategy much is made of near doubles, for example, and choosing to use a near double must be on the basis of recognising that the numbers are next or near to one another. In other words, if the direct teaching of strategies is to operate successfully, then the calculation method to be used should be a function of characteristics of the problem. In the Numeracy Strategy there seems to be an assumption that acquisition of the strategies in itself equips the child to make good strategic choices, but as mentioned earlier some teaching approaches (e.g. Clarke 1997) have an additional strand, of discussing with the children how to choose an appropriate strategy. Either way I find the notion problematic.

The first issue here is ambiguity. For example, the Numeracy Strategy suggests specific methods to be taught addition at Year 2, including:

1. adjusting doubles when the numbers are one away from doubles;

ii. rounding 9s or 11 s to 10 and then adjusting the answer by one; and iii.

partitioning into 5 and a bit when adding 6,7,8 or 9.

The trouble with moving onto the second step, how to choose a suitable strategy, is that 8+9, for example, fits all three templates. Which then is the 'right' method:

i. to adjust a double? (8+8= 16 so it is one more, 17);

ii. to round the nine to ten and adjust? (8+ 1 0 = 18, so it is one less, 17);

iii. or to partition to 5 and a bit? (5 + 3 + 5 + 4 rearranged to get 5 + 5 + 3 + 4 which is 10 + 7, 17) If examining the characteristics of the problem leaves an ambiguity about what to do, what use is examining the characteristics of the problem?

The next issue is the matter of scope. With small number problems there are relatively few examples, but when the numbers become larger, in two digit addition for example, there are literally thousands of different problems. An approach which decides the strategy on the basis of recognition of a type relies on there being clearly identifiable types, and there are perhaps some (those where one addend is in the high nineties, for example) but what about the rest?

Teaching strategies in a direct way, and discussing how to choose appropriate strategies, will lead to an expectation in the children that there is a way of deciding what to do on the basis of an examination of the characteristics of the problem. That is not helpful if it only actually applies to a small proportion of the calculations one might face.

In this respect it might be argued that there is a 'best' method for most problems, for example that 46 + 68 is better suited to a 'partial sum' method than is 46 + 58, for which a bridging through ten approach would be better. But how can one say that in advance, as a result of some recognition of features of the problem? It is a post-hoc argument, based on what is actually easier to do.

Even when the characteristics of the problem allow the identification of a feature that is associated with a particular strategy, that does not mean that it should be your choice. For example one might recognise that 473 + 479 is a "near double" problem, but it would be inadvisable to go that way.

A related issue concerns the execution of the strategy, in that it is possible to choose a method but then not be able to actually carry it out. For example a child may decide to use a "cumulative sum" strategy on 57 + 46 by first adding 50 to 46 then adding 7, but then not be able to complete because he or she cannot add 7 to 96. In the context of a particular problem with particular numbers what works for one child does not work for another, and a decision to follow a particular strategy on the basis of characteristics of the problem does not allow for that. This can be illustrated further by looking at another example of children's "strategies":

13 x

7 "12 times 7 is 84 then add another 7 and get 91 "

"I did 7 times 1 0 then timesed 7 again by 3"

"11 times 7 is 77, then 2 times 7 is 14.77 add 14 is 91."

"First I did 13 times 2 is 26. Then I added, two 26s are 52, so that is four l3s, then added another 26 makes 78 then added a 13 equals 91."

Why did one child use 12 sevens where another used 10 and another II? One factor in it must have been what each child knew. The first approach based on twelve sevens is in a sense easier, but relies on that particular piece of knowledge. In other words, what you do depends on what you know, not just on the characteristics ofthe problem. It seems unlikely that the decision about what to do begins with a review of relevant knowledge. Without that element however, the choice of strategy can come unstuck.

These difficulties with the 'logic' of strategic choice in the context of mental calculation lead me to suggest that we do not calculate in that way anyway; that, away from the contrivances of the classroom, calculation is not 'strategic'. Look at a further example.

57 divided by 3

"3 times 10 is 30. Double 30 take away 3 is 57. 2 times 10 is 20, times 3 is 60.20 threes take away 3 is 19 threes."

"9 times 3 is 27, add one on is 30, so it is 19."

"3 divided by 5 is 1 and 3 divided by 27 is 9 so it is 10 and 9, 19."

"First I saw three 15s that then there was 12 left so 12 share between 3 equals 4 and 4 add 15 equals 19."

There *is* something strategic in the overall process, in that there is a decision to work this way with numbers rather than counting or making a mental sum. Children can also be seen to be strategic for example when they work problems in one way for a teacher who rewards speed over accuracy, and a different way for a teacher who rewards accuracy over speed. The issue is not whether calculation can be strategic, it is whether a child's doing this "strategy" rather than another was strategic, i.e. arose from a prior decision. Did he or she do this or that as the implementation of a choice between perceived alternatives of this kind? Did he or she in any sense review the options and then decide which was best before carrying it through? If so, then the teaching approach described makes good sense. If not, it should be reviewed.

Of course it is very difficult to say for sure that he or she did or didn't.
If you ask the children themselves they just look at you.
Adults, who seem to use similar approaches, by and large deny the strategic element, in terms of how it feels. Rather than a strategic 'decide what to do and then do it' it seems to be much more exploratory, a matter of feeling your way and looking at the numbers and trying this and that until something clicks and it is done *(not* 'something clicks and then you do it').
In engaging with a problem I am not searching for a method, I am searching for a solution.
I work with the numbers, not in an analytic, standing back, reviewing the options, way, but actively, actually combining, comparing, using parts of the numbers or what they are near to, and making progress until I arrive at the answer.

It is fair to say here that the absence of a strategic element in adults does not absolutely confirm its absence in children. It is possible that the automatisation of mental processing that occurs - one aspect of the so called "novice - expert" shift (Ashcraft 1990) - might represent in this case a strategic beginning in children which then becomes unconscious. However I do not believe that this is the case, and that it is broadly similar for children.

A further point of some significance is that mental calculation is *as if* strategic, in that when I have done a calculation and look back over what I did, I can describe it in terms that would also describe a set of actions that had indeed arisen from a prior decision. I can characterise the sequence of actions and give a name to a kind of approach that, if our brains worked differently, could be learned and applied planfully, through choices, that would be strategic. But that is not how it actually happens, in my view. It can seem to be strategic, but in practice is not.
The implication of "strategy" implicit in the teaching approach being considered is that the calculation method used to answer a given problem arises from a strategic decision, a decision to take a particular path when there are alternatives. Yet the use of the term "strategy" to refer to children's (or adults') mental calculation methods need not have this implication. Most teachers would not suggest at first that there is a strategic implication in their use ofthe term. "Strategy" in mathematics education has up to now largely been a convenient term to refer to the mental process by which a calculation is carried out, without considering whether it is done on the basis of a prior decision or not. It is not that the strategic element has been denied, it is just that we did not think about it that much. Our purpose in using the word had been different.
In fact the strategic implication of "strategy" is not universally applied even in cognitive psychology. Ashcraft (1990) for example chooses to use the term "strategy" as a convenient way to

refer to any mental processes that serve a goal related purpose, without any 'strategic' implications. (His is the point that automatisation can obscure the issue.) However the teaching strategy described above strongly suggests that the implication is there in this case. Direct instruction in "strategies" contains an assumption that there is something strategic in mental calculation that has not previously been there in the use of the term, and in my view is not well founded.

The process of finding a calculation approach is not the result of an analysis of types and matching them to pre-learned strategies. It is something to do with how you 'see' the numbers when you look at them on this occasion. You may 'see' one or both of the numbers as a composite of different parts, perhaps in different ways. You may 'see' the proximity of one or both of the numbers to other numbers, like in rounding. If one of the ways in which you 'see' the numbers connects with knowledge that you have, then you begin to work it out in that way, based on an awareness of what can be inferred - how the numbers can be changed and used. If it doesn't work out, you start again and try a different way in. Children cannot be trained to see future events in a particular way, other than in a very narrow range. They can be encouraged to look for the characteristics of numbers, and so on, but to say "Look to see ifthey are near doubles" is doing them no favours. We want children to have success in a wide range of calculations, not just the happy few that happen to fit with 'neat' strategies for which they have the requisite supporting knowledge. A mechanistic approach to teaching strategies is therefore not to be recommended. However, all I would want to change is teaching the methods without the implication that mental calculation is a matter of matching procedures to requirements to teach "strategies", but as possibilities, with lots of opportunities for children to find their own way through number challenges in an atmosphere of invention not correct choices. Ironically, when I have seen the Numeracy Project in action, that is how it is - pupil sensitive, eliciting, development oriented teaching. The rhetoric of the Numeracy Strategy, however, has more of an acquisition and application flavour, and my worry is that in general use the teaching approach adopted will be that outlined above, which I fear would inhibit the improvement in standards that is so earnestly sought.

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