VISUALISATION AND USING TECHNOLOGY IN A LEVEL MATHEMATICS

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This project seeks to identify and evaluate the ways in which existing technology can be utilised to promote and develop students' powers of visual is at ion and to encourage the usage of these skills in A level mathematics lessons. At present, a small scale pilot study has been carried out and the resulting data has been analysed. After briefly summa rising current research in the area of visual is at ion and technology in this paper, I will report on the findings of this initial pilot study, in which materials developed for use with the TI-92, aimed at promoting students' abilities to visualise the graphs of functions, were trialled with a class of thirteen year twelve students. The subsequent consequences for future directions of the research will also be discussed.

Introduction

Visualisation is increasingly being accepted as an important aspect of mathematical reasoning. Studies have revealed that 'activities encouraging the construction of images can greatly enhance mathematics learning' (Wheatley and Brown, 1994). Indeed, potentially, technology could assume a very powerful and influential role in stimulating and shaping students' powers of visual is at ion, and as such may prove to contribute significantly to the depth of students' understanding.

Zimmerman and Cunningham (1991) insist that mathematical visua1isation is not merely 'math appreciation through pictures' - a superficial substitute for understanding. Rather they maintain that visualisation supplies depth and meaning to understanding, serving as a reliable guide to problem solving, and inspiring creative discoveries. In order to achieve this understanding, however, they propose that visualisation cannot be isolated from the rest of mathematics, implying that symbolical, numerical and visual representations of ideas must be formulated and connected. This project is conceptualised on the basis that visual thinking and graphical representation must be linked to other modes of mathematical thinking and other forms of representation (Tall, 1989).

Issues Surrounding Visualisation

Within the current literature there exist many differing notions of the key terms associated with the area of visualisation in the learning of mathematics, each developed with respect to a specific research purpose/focus, and each drawing on and expanding previous ideas.

Mariottii and Pesci (1994) acknowledge *visualisation* occurring when 'thinking is spontaneously accompanied and supported by images'. Mason (1992) regards *visualising* as 'making the unseen visible' and *imagery* as 'the power to imagine the possible and the impossible'. Solano and Presmeg (1995) see *visualisation* as 'the relationship between images' - 'in order to visualise there is a need to create many images to construct relationships that will facilitate visualisation and reasoning'. Hitt Espinosa (1997) suggests that *visualisation* of mathematical concepts is 'not a trivial cognitive activity: to visualise is not the same as to see'. To *visualise* is the 'ability to create rich, mental images which the individual can manipulate in his mind, rehearse different representations of the concept and, if necessary, use paper or a computer screen to express the idea in question'.

Unfortunately, despite the current views of researchers surrounding the importance of visualisation, there is still a tendency for visualisation to be undervalued in mathematics classrooms and consequently some students, whilst able to visualise mathematically, often opt for non-visual, more 'conventional' approaches to problem solving (Presmeg, 1995). Traditionally, a greater emphasis has been placed on algebraic or analytic proof, despite the proposed legitimacy of visual theorems. Presmeg's findings (1986) indicate that an ability to apply and interchange both visual and non-visual methods in problem solving is particularly advantageous for students, especially where one mode is more appropriate. However, the teaching of school mathematics is predominately non-visual and 'visualisers are seriously under-represented amongst high mathematical achievers' (ibid).

Although, images presented to students by teachers will influence the students' understanding and individual construction of such images, the students' conception of these images will not necessarily correspond to that of the teachers' (Mason, 1992). Indeed, 'visual ideas often considered intuitive by an experienced mathematician are not necessarily intuitive to an inexperienced student' (Tall, 1991). Students should be encouraged to create and explore their own images (Cunningham 1994) - a visual understanding of a given situation is more robust and is thus more likely to be remembered by the student in the longer term than a purely algebraic proof. Yet, Presmeg (1986) outlines four particular difficulties involving imagery; images/diagrams viewed inappropriately, inflexible thinking when dealing with a non-standard diagram, rigid uncontrollable images and vague imagery. She (ibid), also, suggests that 'less imagery is used with greater experience or learning'.

Visualisation skills may be employed by students privately to clarify, interpret and make sense of the given problem intuitively, as tools for 'meaning-making' (Wheatley and Brown, 1994) although, such processes are unlikely to be explicit in written arguments (Presmeg, 1995). Furthermore, the usage of visual techniques is comparatively time intensive suggesting that tests and examinations will tend to favour the non-visual thinker (Presmeg, 1986). In addition, visual thinking requires nonsequential, parallel processing of information, and as such poses a greater cognitive challenge to students than step by step sequential algorithmic reasoning (Eisenberg and Dreyfus, 1991).

The Perceived Role of Technology

In light of the recent advancements in technology, a whole range of computer programmes and scientific instruments are currently available with the potential to assist students in the formation of visual mathematical images. One of the main objectives of this research is to evaluate and develop materials and strategies which aim, as far as possible, to maximise this potential, with particular emphasis on the graphical calculator. However, the role of the computer in this respect is, also, regarded by the researcher as extremely important and influential and is thus explored in this review of current literature. Overall, the findings of studies involving graphical calculators appear to be very similar to those which utilised computer technology, although the similarities and differences between these two types of technologies should not be overlooked.

Many researchers realise the potential of utilising technology to promote and encourage visualisation skills (Souza and Borba, 1995; Smart, 1995). In particular, computer based visual approaches in teaching mathematics can i) increase motivation and ii) provide an opportunity to pursue an alternative and yet complimentary mode of thought to the traditional symbolic approach (Cunningham, 1994). Technology can be utilised to enable students to develop a deeper insight into the relationship between functions and their graphs (Carulla and Gomez, 1997). Furthermore, technology can be particularly useful in exploratory learning, where students are able to formulate concepts for themselves and benefit from visualisation in the process (Tall, 1991).

However, despite the advantages, students may still misunderstand, misinterpret and therefore misuse information provided by graphic calculators (Carulla and Gomez, 1997). Furthermore, such technology may encourage students to focus primarily on graphical representation whilst neglecting other modes (ibid). In contrast, other researchers report that graphical calculators can be utilised to foster the transitions between and exploration of different modes of representation (Ruthven, 1990). Multi-representational software, however, could contribute towards misunderstanding and confusion amongst students; any difficulty experienced with one particular representation could be intensified by the presence of other forms of representations (O'Reilly et al,1997). There is a danger that students could become 'saturated by images' (Mason, 1992). Alternatively, students may become too dependent on technology, regarding the solutions generated as irrefutable (Smart, 1995). Zimmerman and Cunningham (1991) believe that certain fundamental visualisation skills are prerequisite for meaningful computer based visualisation.

Many researchers maintain that the use of technology can promote collaborative learning and equal opportunities (Smart, 1995). In particular, female students, have benefited from the private nature of the graphics calculator (ibid). Ruthven (1990), also, found that a reduction in student uncertainty and anxiety accompanied regular use of the graphics calculator, and hence stimulated improvement in the 'confidence, competence and performance' of all students, especially that of the females. This study will investigate how graphical calculators affect the visualisation capabilities of the female students in comparison with the males, with the aim of determining whether female students benefit, in this manner, to a greater or lesser extent.

<u>The First Pilot Study</u>

Initial classroom trials were carried out at a school in Sheffield for a period of six hours during February 1998, with a group of five male and eight female year twelve students. The fieldwork involved participant observation and a post-trial questionnaire. Each individual student was given a TI-92, although they generally worked together in pairs, sharing ideas. The exercises featured graphing functions, and involved exploring and identifying the effects of transformations, finding inverse functions, solving equations - graphically and algebraically, and investigating trigonometric and logarithmic identities. The main aim of this pilot study was to enable the researcher to assess the suitability of early materials and techniques, to elicit preliminary reactions to the use of technology and to provide a framework for further data collection.

Student Questionnaire Responses

The questionnaire responses indicated that, generally, this particular group of students viewed technology as an important addition to the A level mathematics classroom - a quick and accurate means of strengthening their understanding and visualisations of functions. However, some feared over-dependency and accompanying laziness, but would nevertheless welcome further use of technology in the future. Thus, these students appeared to appreciate the opportunity to use the TI-92 and seem to have benefited mathematically from the experience.

The Students' Work

A preliminary examination of the students' work revealed that a high proportion of students often assumed that the TI-92 was displaying the whole graph without using the zoom in and out facilities. The function x^2- x^3 caused particular problems. The students were asked to sketch the graph of the function and to determine the nature and co-ordinates of any turning points. The first graph (fig I) is drawn using ZoomStd (where the x and y axes vary from -10 to 10, in divisions of I unit), and the second graph (fig 2) results from zooming in on the first to a degree of factor six, centred on the origin. The second graph provides a much better picture of the actual shape of the graph. Yet, all of the students who attempted this question failed to use the zoom facilities and thus drew a sketch of the function which resembled fig 1. Consequently, they mistook the point (0,0) as a point of inflection (clearly a local minimum in fig 2) and were unaware that a local maximum existed and as no-one checked their results by differentiation these errors were undetected. Clearly, these turning points were missed because they were not initially visible on screen.

fig 1

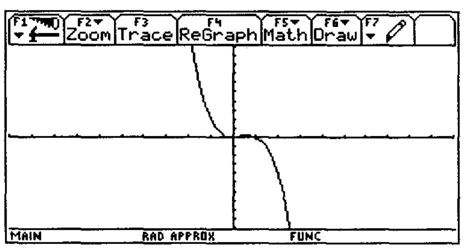
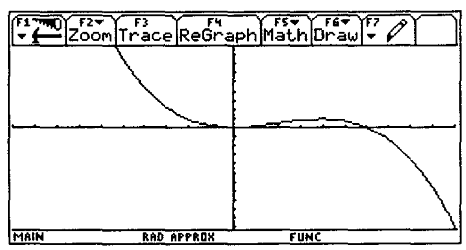


fig 2



In contrast, some students believed that the graph of y = (x+ 1)/(x+2)2 had a minimum turning point at x = -2. These students failed to realise that the function is undefined at this x value as they completely misinterpreted the graphs displayed by the TI-92 and neglected to inspect the equation.

fig 3

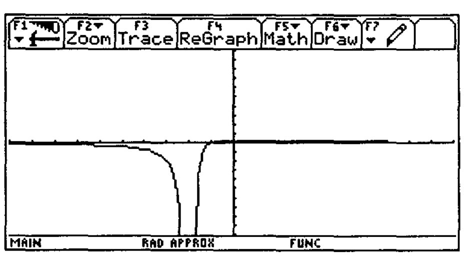


fig 4

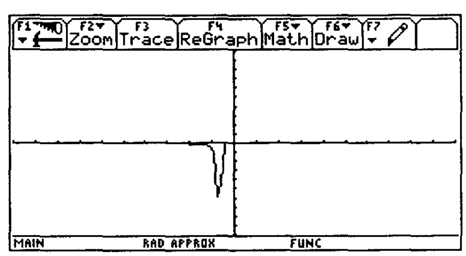


Fig 3 shows the graph of y = (x+ 1)/(x+2)2, using ZoomStd again, whereas fig 4 is obtained by zooming out on the original graph, centred on the origin, by a factor of three. Since only part of the graph appeared to be visible on the screen in ZoomStd, some students choose to zoom out, producing graphs resembling fig 4, which seemed to have a minimum stationary point, and so these particular students (who were using the zoom facilities) were fooled. As before, these students did not spend time thinking logically about the function or picturing what the function might look like for themselves - they were confident that the technology provided them with the correct answer. Smart's (1995) research emphasises this problem, referred to as the 'magic:' element of technology.

In addition, few students successfully completed the algebraic components of certain questions, and fewer still actually specified the symbolic form of the graphs resulting from a series of successive transformations, even though this was requested. Thus, these students tended to concentrate on graphical representation in questions involving both graphical and algebraic aspects. However, the remaining questions were completed satisfactorily. In particular, eleven students were able to identify the graphs of all six functions in the final exercise, which was an encouraging outcome.

Implications for Future Data Collection

The data collected in this pilot study has enabled the first evaluation of the classroom materials and approaches devised by the researcher, aimed at promoting the development of student's powers of visualisation using technology to be undertaken, thereby permitting some initial progress in terms of achieving the second objective of the research. However, there was insufficient data to provide notable insight into the third and fourth objectives; to investigate the ways in which the technology acts as a tool in mediating the development of students' powers of visualisation and to investigate how powers of visualisation might be evoked and be developed by the use of mathematical software. To what extent did the materials encourage, if at all, visual thinking?

Thus, preliminary results suggest that it would be useful to establish a means which would indicate how students initially approach problems involving functions. In other words, before the research takes place, do they adopt a predominately visual, algebraic or numeric approach? If their approach tends to be visual, how successful are they? If their approach is not visual, how do they perform when asked to work visually? Moreover, following the introduction of technology does their preferred mode of operation change? Do the visualisation skills of all students (not only those who prefer to work visually) improve? Do all students necessarily have a visual approach? Teachers will be interviewed to determine the extent to which they have used visual methods in their teaching of functions, and in lessons generally. It is, also, recognised that to try to distinguish between visualisers and non-visualisers is problematic; there is a continuum between students who can be regarded as almost entirely visual thinkers and those who are virtually exclusively non-visual and furthermore there are few students at the extremes - for different types of problems individual students may use different methods of solution. In the future pre-trial exercises, questionnaires and interviews will be utilised in an attempt to ascertain answers to these central research questions.

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