ADVANCED MATHEMATICAL THINKING WORKING GROUP

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ABSTRACT: Initially, this group will want to assess the interest in Advanced Mathematical Thinking (AMT), with the possible aim of creating links with other international working groups in the field (e.g. P ME). The group will be introduced to some of the major issues in AMT research today, and possible future developments. Themes of interest at the moment include: students' conceptions offunction, limits, proof, and students' conceptions of differentiation and integration. It will look at the relevance of social, pedagogical and methodological issues, such as the use of algebraic/geometric software or methods for analysing (meta-)cognitive behaviour, in developing a more suitable model of AMT. It is a major aim that the group will not concentrate on purely psychological issues but will discuss the nature of mathematics, per se.

The group met for the first time and a keen interest was shown by a dozen or so people in the field of Advanced Mathematical Thinking (AMT). The group was introduced to some of the main ideas of AMT and some of the main epistemological notIons which have been developed and criticised these past 18 years.

We initially discussed what AMT was and what was known so far. AMT was said to be a branch of the Psychology of Mathematics Education but fundamental questions about the nature of mathematical thinking and advanced problem-solving processes were asked. This was for the purposes of dealing with problematic issues in students' problems-solving process, epistemological obstacles (Sirepinska 1987) and pedagogy (Dubinsky & Tall, 1991).

The group was asked to think about possible discrepancies in a psychology of mathematics education and how we could address these issues. Many in the group were concerned with the word "advanced" and what it referred to; whether it was exclusive or elitist. We looked at parts of Tall (1991) which related advanced mathematical thinking to elementary mathematical thinking., and that the *advanced* did not always refer to advanced mathematics but advanced thinking.

Analogies between elementary axiomatic learning and advanced mathematical

(under-) graduate thinking are represented in Tall (1991; Ch. 15) and Gray & Tall (I 994), for example.

Primarily, we are concerned with advanced mathematics, since higher-order thinking is prominent in advanced mathematical problem-solving, but research in elementary thinking can highlight epistemological notions and ontological ideas in the former. How a student might conceptualise a function and manipulate functional notions; how a student might manipulate function in an ineffective way might well be to do with elementary functional work in axiomatic relationships.

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The working group concentrated on two themes in the first session:

I. What has AMT offered to Mathematics Education?

2. How can we re-shape AMT (i.e. does the group have a purpose)?

Part I: What has AMT offered to Mathematics Education?

It is evident in a review of the literature (Hegedus, 1998) that AMT has offered us two things:

- An epistemological framework of rigorous psycho-mathematical concepts which have addressed thinking in an advanced mathematical domain far more intuitively than some of the vague generic psychological terminology used in educational research to date.
- Data to increase our understanding of students' problem-solving behaviour. These have generally been incorporated into technological pedagogical packages.

The main epistemological notions which have been developed/discussed in the AMT literature are:

1. Concept Image/Concept Definition.
2.Reflective Abstraction. 3.
Procept.

The notions were explained to the group and examples were shown. The group responded with considerable interest to the final notion, the procept, and it was agreed to discuss this term in greater detail at the next meeting.

Examples form the Calculus (re Limits and function) and didactical engineering were extracted from Tall (1991; Chps. 9,10,11) and discussed. Some of the main epistemological obstacles discussed were:

- The failure to link geometry with numbers.
- The notion of the infinitely large with infinitely small.
- The metaphysical aspect of the notion of limit. • Is the limit attained or not?

Part II: How can we re-shape AMT?

The group discussed the position of AMT as a research program and a school of psychological thought. The group adhered to ideas that elementary mathematical thinking could be examined in an AMT paradigm to relax the elitist nature that is inherent in the body of thought and which makes it possibly unapproachable for certain people.

The major theme was on acquiring an epistemological base with which we are confident with and with which we can discuss. From this base we might develop, deconstruct, or re-construct epistemological notions in the AMT paradigm and work towards a more dynamic epistemology and suitable methodologies to create a better research environment.

The group believes that we need to examine what ideas have been put forward so far about the nature of Advanced Mathematical Thinking, to critically analyse them, and

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to work towards a clearer understanding of the nature of advanced mathematical thinking.

I put forward three issues as momentum for the construction of a new psychology of AMT.

- The existence of a dichotomy between traditional psychology and educational psychology.
- Psycho-linguistic vs. linguistics. Do we have some un-entangling to do?
- Generic psychological terms are too abundant. We need to analyse them and synthesise a formal psycho-mathematical discourse which is useful to the mathematics and mathematics educational community.

The group concluded with an aim to discuss the procept in further detail at the next meeting. Developments of the group and AMT resources are housed on the web at http://www.soton.ac. uk/~amt.

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