TEACHER TRAINEE STUDENTS' UNDERSTANDING OF OPERATION SIGNS

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Abstract

Following the invention of operation signs by young children working m *a tramj'ormationally-focussed problem scenario, a similar transfonnational model was presented to 4th year teacher trainee students in which they also were asked to invent signs. The signs referred to the strategies of counting-on, counting-back and counting-up. The students' responses and their understanding of the difference between counting-back and counting-up will be discussed, and participants invited to give their own opinions on the mathematical status of such 'counting signs '.*

Introduction:

The essential role of symbols in the development of mathematics in general and arithmetic in particular is unquestionable (Kaput, 1987, Sfard, 1994, Pratt & Garton, 1993). **In** the classroom, collections of objects are symbolised by number symbols which are manipulated according to rules. Because these rules correspond to how collections of objects behave in the reality, predictions can be made about the size of collections when these collections are added-to or taken-from etc. For example, 17 - 3 can describe the action of taken 3 objects from 17 objects. Augmenting this expression with = 14 displays the result of that action. When this result is omitted, as in '17 - 3 =' , the expression is conventionally taken as a question (17 - 3 = ?). **In** the context of school computations, this representedsituation can be realised or re-invoked by the child using concrete objects to model the expression and in this way, the result can be found experimentally. **In** this case the expression 17 - 3 =? takes the form of an *instruction* to perform the action necessary to find the answer. This dual function of description and instruction causes problems with the interpretaion of signs as the following investigation shows.

Background to the investigation

In an earlier experimental teaching situation dealing with stepping stones (Womack, 1998),5 year-old children carried out instructions and answered questions within a transformational framework. Results showed that they could confidently use signs which indicated how far they were to count-on or count-back along sequences of numbers. **In** a slightly different context they could also count-up the number of steps between two given letters or numbers (See Figure 1).

Counting procedures are not normally taught explicitly but are nevertheless necessary In order to compute mentally. **It** was of interest to know how far the intuitive skills of young children could be made explicit to teacher-trainee students when both types of sign were used in the same problem-context. The signs invented by the children (Note 1) in the earlier experiment (see above) were sloping arrows which instructed the sign-user to step-on (or count-on) and step-back (or count-back). A *different* arrow sign was used to indicate that the number of steps between two stones was to be counted.

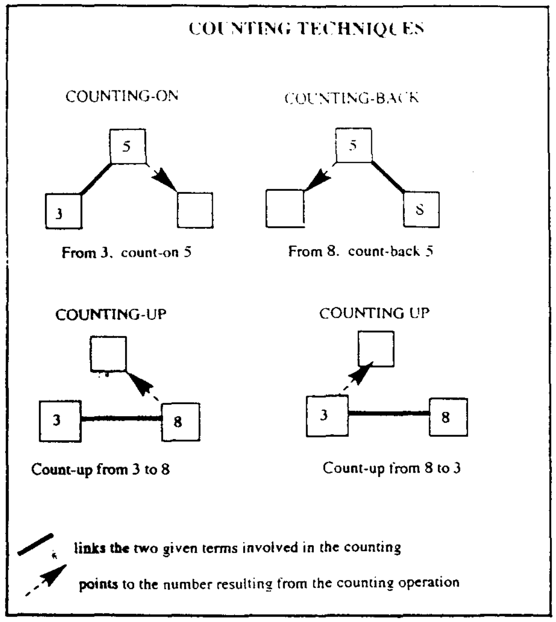

Fig. 1

With the 5 year old children, the use of this latter counting-up sign was NOT used at the same time as the counting-back sign (to indicate counting-back was to be carried out), since it was felt there was a possibility for confusion.

The stepping stone model used with children was initially based on the standard Oksapmin numeration system (Note 2) outlined by Saxe (1982), in which to count, one begins with the thumb on one hand and enumerates 27 places around the upper periphery of the body, ending on the little finger of the opposite hand. In the investigation described here, a *letter sequence* was used, to avoid possible confusion of numbers representing both positions and movements. The isomorphism between movement from body position to body position and movement from one letter to another is shown in Figure 2.

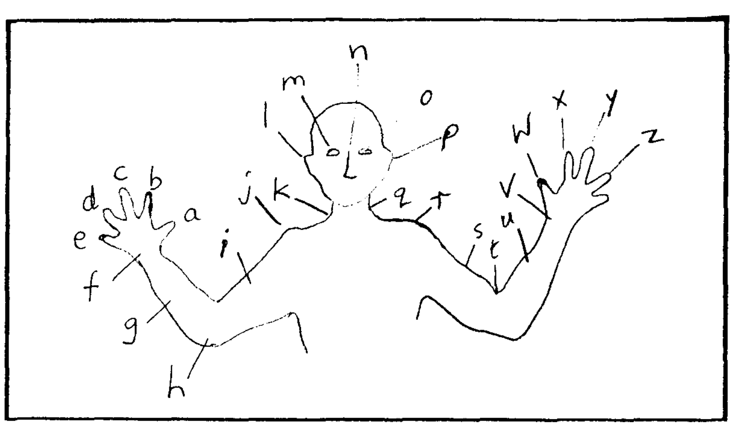
Fig. 2

The Investigation

The following question sheet (Figure 3) using an alphabetic sequence model was given to nine 4th year teacher trainee students for completion in about ten minutes time in an 'open class' situation .

A B C D E F G H I J K L M N O P Q

To reach any letter from an adjacent letter, we '**jump**'. Moving Left to right is called 'jumping-ON'. Moving Right to Left is called 'jumping-BACK'.
e.g. From B, jump-on 2 ...... reaches D From B, jump-back 1 ..... reaches A

Question1 : From D, jump-on 7. Which letter do you reach?
Question 2 : From D, jump-back 3. Which letter do you reach?
Question 3 : From E, jump (on) to K. How many jumps is that?
Question 4 : From G, jump (back) to B. How many jumps is that?

If you wanted to put these questions into a symbolic form *which did not use words,* how would you do it? What symbols would you use to ask Questions 1, 2, 3, & 4?

Fig.3

The results from all the students were then discussed in an open forum. A further question (Question 5) asked for an *explicit* symbolic formulation for these four broad questions. This required an expression in which the unknown (asked for) term was expressed explicitly by means of a relation sign linking the two given terms (letters in this case).

Summary of Results

These are shown in Figure 4. The most common response of the students was to use an arrow. However, in some cases the arrow pointed from one letter to another - indicating that the number of jumps between the letters had to be found; in other cases it pointed from a letter to a number, instructing the reader to count on that many jumps from the given letter. (In these cases, the question asked was *assumed* to be, 'Where does such an action take you?') Note also that in Question 4, the order of letters given in the written question is the reverse of their order in the alphabet.

Questions 1 and 2: In question 1, all students drew an arrow from a letter, indicating perhaps that their thinking began with a letter. Student 5 began with a letter, but began the arrow from the number (i.e. from D, 7 forward). All students used arrows as signs.

4 students (nos. 1,2,3 & 4) used signs and number in parallel. Some used a box instead of the equals sign. 1 student used the sign *after* the number. 3 students used the sign *before* the number. 1 student used conventional signs before the number.

Question 2: The arrow in each case indicated the direction of the movement along the letter sequence and students 1 to 4 indicated the 'missing' letters by a square placed in its correct sequential position (to the left of D).Their representation therefore referred to the alphabet model rather than the written words. Students 6, 7 and 8 seemed to be trying to say 'From D jump-back 3' using symbols in the same order as the written words.

Questions 3 & 4: 2 students (nos. 1 & 2) recognised that this was the converse question to that in Questions 1 and 2. Therefore they put arrow and missing 'jump-on/back' box again in parallel. 3 students (5, 6 & 8) used the same form of arrow sign to mean both 'jumpon/back' (in Questions 1 and 2) and 'jump-on/back *to'* (in Questions 3 and 4). 1 student didn't make clear what the question was. 1 student used conventional signs - but not for conventional aggregation but to indicate the number of jumps to be made.

Question 4: 2 students (3 & 4) used the same logical system as students 1 & 2, but the expression required reading in the reverse order (B/G). Their arrows indicated the relative position of the given questions.

Question 5: Attempts to pose Qs 3,4 'explicitly' was not achieved successfully except by giving signs a dual interpretation. That is, using the arrow to mean 'jump-on 7' as in 'D ---> 7, but also to mean 'jump-on TO 7' (E ---> K.) Only students 7, 8 & 9 attempted to make the unknown the subject of the expression. Student 7 answered questions 1 & 2 explicitly, but not Questions 3 & 4. Student 8 answered Questions 1, 2, 3 & 4 explicitly, but used the same sign in all questions.

Fig. 4

Student	Q1: From D jump-on 7	Q2: From D jump-back 3	Q3: From E jump to K	Q4: From G jump (back) to
1	D —7→ □	□ ←3— D	E —□→ K	G ←□— B
2	D —7→ □	□ ←3— D	E —□→ K	G ←□— B
3	D ⌒7→ □	□ ←3⌒ D	E ⌒□→ K	B ←□⌒ G
4	D —7→ □	□ ←3— D	E —6→ K =	B ←5— G
5	D 7 ↦ =	↤ 3 D	E ↦ K =	G ↤ B =
6	D → 7 =	D ← 3 =	E → K =	G ← B =
7	D ⇉ 7 D → 7 = □	D ⇇ 3 D ← 3 = □	E ⇉ K E → □ = K	G ⇇ B G ← □ = B
8	D → 7 = ___	D ← 3 =	E → K = ___ E → ___ = K	G ← B = ___ G ← ___ = B
9	D + 7 = D → 7	D − 3 =	E + □ = K	G − □ = B

Summary of student views

In discussion, students were *uncertain* whether they were using the same mental process to find a *letter* and to find a *number*. Therefore, if letters were to be replaced by numbers - as in the conventional counting procedure, clearly there would be a great potential for confusion. Some students argued that similar signs could be used to refer to similar processes (such as jump-on and jump-back) but should not be used for different processes. Others argued that two different mental processes might be represented by the same sign, provided the difference was understood from the context.

Discussion of explicit and implicit signs

The sign required for questions 1 and 2 represents an action necessary to perform in order to answer the question 'Which letter do you reach?' In conventional arithmetic this would correspond to the 'take-away' sign in the conventional expression 17 - 3. However, the sign required for questions 3 and 4 is a *meta-sign,* in that its function is to ask the reader to find another sign. This meta-sign represents the action required to find another action - that needed to move from one given letter to another letter - a number of jumps. For example, in conventional arithmetic, given an initial number (say 11), and a final number (say 14), the child is require to find the amount added - that is 11 + ? = 14. To put this in *explicit* form, it is required to state what operation must be carried out with the numbers 11 and 14 in order to generate the answer in the explicit form. Conventionally, this operation is again one of subtraction: i.e. 14 - 11 (the difference between 11 and 14). Note however, that the operation required (and found) is 'add 3'; whereas the *meta-operation* required to generate this (former) operation is conventionally known as subtraction. Note that transformational arithmetic distinguishes the operation which reverses the adding-on action, from the metaoperation used to find what that adding-on action is/was (Note 3).

Implications

The following are important reasons for studying counting signs:-

1. There is a need to signify the basic counting techniques of count-onlback and count-up; without them we cannot conveniently instruct children to adopt a certain strategy. Nor can children record which of these activities they carried out. This is clearly a disadvantage when attempting to develop a pedagogy for teaching counting skills.

2. Counting signs could be used to describe the counting strategies of non-literate cultural societies such as the the Kpelle of Liberia (Gay & Cole (1967).

3. Since counting appears to be an intuitive strategy adopted by children prior to formal instruction, it would be of interest to psychologists and mathematicians to investigate the nature and properties of such signs.

Notes:

1. The 'counting signs' are currently being trialled in a National Numeracy Project school.
2. The counting procedures used by the Oksapmin involved what we could call a 'pre-counting-up' strategy in which the the names of the body parts or positions between two given positions were listed (orally) rather than counted. A more usual counting-up procedure is to number these 'counted between' body parts by calling them by other body parts. This is the standard procedure for counting-up between say 7 and II by enumerating 8,9, 10, 11 giving a count-up of 4.]

In the model, counting-up from H to M is equivalent to finding how many letter-jumps between H and M. This is achieved by pointing to I, J, K, L, M and matching the pointing to the letters A, B, C, D, E . (I.e. since E is the 5 th letter, there are 5 letter-jumps between I and M.)

3. A fuller explanation of 'transformational arithmetic' is in preparation.

References

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