Possibilities in Peirce's Existential Graphs for Logic Education

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In our experience students find learning logic difficult, this view is supported in the literature. Charles Sanders Peirce, logician, semiotic ian, teacher, envisaged a representation that would provide tools to enable everyone to reason with formal logic. To this end, and basing his work on his own semiotic principles, he developed a system of graphical reasoning. In this session we will present Peirce existential graphs and consider this form of reasoning as providing possibilities for improving logic teaching and learning. We will also outline our plans for a future study to evaluate the Percian approach.

Logic, Mathematics, Education

> "Now although a man needs not the theory of a method in order to apply it as it has been applied already, yet in order to adapt to his own science the method of another with which he is less familiar, and to properly modify it so as to suit it to its new use, an acquaintance with the principles upon which it depends will be of the greatest benefit. For that sort of work a man needs to be more than a mere specialist; he needs such a general training of his mind, and such knowledge as shall show him how to make his powers most effective in a new direction. That knowledge is Logic. "(Peirce 1931-1958, 2.67)

> "much of the area of logic is not easy to comprehend ,", with practice and concentrated effort one can achieve a useful understanding" (Kelly 1997)

Logic is the backbone of mathematics and computer science, yet it is clear from the limited research in this area that many intending mathematicians and computer scientists have very limited logical facility. For example the recent London Mathematical Society report (Howson 1995) suggests that students ability to prove is very poor and Barnard (1995) has shown convincingly that simply negating a statement can be problematic.

Of course logic is not mathematics, and logic education is not mathematics education yet there are such strong connections between logic and mathematics that we feel it appropriate to discuss logic education at a mathematics education conference. Logic has a place in the history of mathematical foundations and in mathematical notation, and today we find that language of mathematical logic from the earliest mathematics classrooms. Gibson (1986) advocates teaching logic at school, and even wants to replace mathematics with logic at this level. His reasons are twofold, the first that logic is more useful than mathematics in everyday terms and the second that anything that is of value in mathematics terms of education is also in logic. We cannot deny the importance of logic in our technologically driven daily lives.Ifwe accept Gibsons' idea however, teaching logic remains beset with problems. Barnard (1995) found, after testing 684 students across all age ranges for their ability to negate statements, that in general whatever the level of formal training students are not very good at such negation. Common errors were the negation of the main verb only *(stay awake* became *fall asleep)* and incorrect negation of the quantifier *(there exists* became *there does not* *exist).* Interestingly he also found that students were influenced by context, those less contextualised questions were answered much better that those that actually made some sense.

We can add, along with other teachers of elementary logic, a wealth of anecdotal evidence to support the claim that there are problems in learning logic. As one example consider the problem of material implication through the "raining" context (box 1) which always causes our students intense problems in terms ofthe truth table representation and the case in which it doesn't and I don't bring my umbrella.No textbook that we have seen explains this adequately. Confusion is compounded by statements such as:

> "One could disagree with the reasoning behind this interpretation but we will avoid philosophical tangles and simply remark that this definition provides a coherent and fruitful element in the formalisation of classical logic" (Kelly, 1997 p.7).

<u>I promise that if it rains tomorrow then I will bring my umbrella</u>

(a) If it rains tomorrow and I bring my umbrella am I keeping to my promise?
(b) If it doesn't rain tomorrow and I bring my umbrella am I keeping to my promise?
(c) If it doesn't rain tomorrow and I don't bring my umbrella am I keeping to my promise?
(d) If it rains tomorrow and I don't bring my umbrella am I keeping to my promise?

box 1

Representations

The reasons for these problems are as yet unknown although there is evidence to suggest that the degree of symbolism and abstraction in logic (Dubinsky, Elterman et al. 1996) and in mathematics in general (Vile and Lerman 1996) cause conceptual difficulties. The problems that Barnard observed with context point towards language, and other choices of representation as problematic and ambiguous. This notion is not new and the philosopher, semiotician and logician C. S. Peirce, as early as 1906, suggested that a more graphical approach to logical reasoning would help to clarify the process. To this end he developed existential graphs which are a graphical representation of predicate logic with the express purpose of logic teaching and clarity in mind (Houser, Roberts et al. 1997). Peirce expressed is view clearly during one of his lectures on logic:

> Our purpose, then, is to study the workings of necessary inference. What we want, in order to do this, is a method of representing diagrammatically any possible set of premisses, this diagram to be such that we can observe the transformation of these premisses into the conclusion by a series of steps each of the utmost possible simplicity.
>
> What we have to do, therefore, is to form a perfectly consistent method of expressing any assertion diagrammatically. The diagram must then evidently be something that we can see and contemplate. (Peirce 1931-1958, 4.429-430)

Peirce apparently always thought in pictures, he attributed this to his left-handedness. He combined his Pragmatism, semiotics and logic into a clear system of diagrammatic thinking that would provide diagrams upon which one could experiment (4.530), much like the way scientists use diagrams in their own work. Traditionally logicians have been critical of the use of diagrams as an aid to logical deduction but Barwise and Etchemendy (1996) support Peirce's view and challenge the "logocentralism" of logic, arguing for the use of diagrams to support reasoning on the grounds that they contain much more knowledge that equivalent forms.

There has been much recent interest in applications of Peirce logic (Lukose, Delugach et al. 1997). Peirce suggested that" [through a] System of diagrammatization .,. any course of thought can be represented with exactitude" (1906, p492) and it is well known, in mathematics at least, that diagrammatic representations aid comprehension (Pimm 1996). Allwein and Barwise (1996) demonstrate the power of a graphical environment in logical reasoning and results in teaching seem positive, although (to our knowledge) this has not been fully investigated through empirical research. It would seem that there is general support for the view that a less abstract approach to logical reasoning would provide possible route to overcoming the identified problems in learning logic. In this short paper we will take a look at the expressive power of Peirce graphs with a view to demonstrating their possibilities for logic teaching

Peirce graphs, a *primer*

We want to suggest that existential graphs are more visual, more holistic, and more transparent to the underlying logic that traditional symbolic representations. Here we will give a simple introduction to Peirce existential graphs, for more details see, (Hammer 1996) or (Polovina and Heaton 1992). Existential graphs consist of two elements a *sheet of assertions,* which is essentially a blank sheet on to which assertions may be placed, and a *cut* which is a ring around an assertion equivalent to ~(...), thereby describing the 'not' relationship, also known as *negative contexts.* Combining assertions and two negative contexts we can form a *scroll,* a basic if then statement. Figure 1 shows a scroll representing p => q .

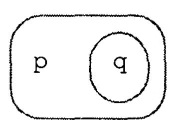


Fig 1

The mere co-existence of each propositional element in existential graphs automatically describes the 'AND' relationship Take for example Modus *Ponens,* represented in traditional propositional logic as (p *n* (p => q)) => q. Existential graphs always represent logic denoted in 'and' and 'not' form, thus the antecedents of *Modus Ponens* can be represented by the first of the graphs in figure 2.

According to Peirce's rules of existential graphs' inference, any dominated element that matches a dominating element may be 'rubbed out' or *deiterated.* As p is dominated by its matching p in the outermost context, it can be deiterated leaving q surrounded by two rings alone. After the removal of the constraining p within the single negative context level, the resulting graph simply states

- (-q) which, by *double negation,* results in q being asserted. We can therefore describe *Modus Ponens* by the following existential graphs:

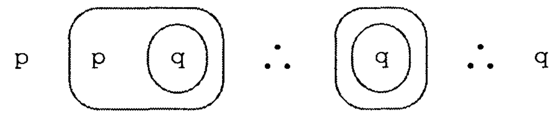


Fig 2

From this example we can begin to see that existential graphs vividly illustrate the contextual interrelationships between the propositional. The above existential graphs reveal that q is asserted once p exists, since q exists in the context of p.

Apart from avoiding the need to learn rote 'rules of inference' from the outset, the visuality that existential graphs offer obviate the requirement to consider truth tables. We can see from the examples thus far that the conjunction of all the graphs, which is automatic as explained earlier, results in true. This consistency is evidently not the case in the following, as deiteration and double negation would indeed confirm:

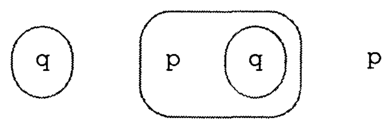
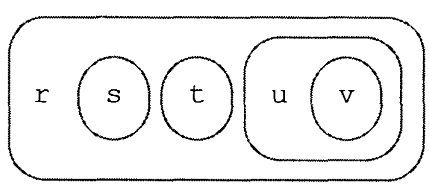
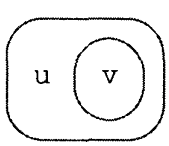


Fig 3

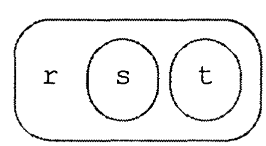
Clearly p and ~q, as well as ~p and q, are inconsistent. Thereby (-q *n* (p ~ q)) ~ p is false. Similarly (p *n* (p ~ q)) ~ -q is also false. All this can be seen to be so without the learner needing to memorise or draw up truth tables, as we have observed, this is a difficult task for students of logic. Of course; we could devise more complex examples that would highlight these advantages. For instance could the learner better tackle problems based around the following interrelationships if presented in traditional propositional logic form?



The truth of the graph: would mean that *all* this



could be deiterated to yield:



Given that the advantages are so self-evident we move on to consider the 'OR' relationship, $x \cup y$, which maps to $\sim(\sim x \cap \sim y)$ i.e.:

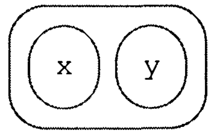


Fig 4

As we would expect - x would result in the deiteration of x and its negative context, thus enabling y to be double negated to release y into the outermost context and thereby be asserted. This also shows us that -x ~ y and, for that matter, -y ~ x. However what would x give us? After deiteration the result would be (fig 5):

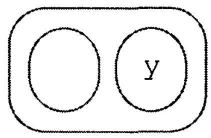


Fig 5

Here y cannot be determined from x as such, though we have shown that no inconsistency arises. As it happens there is a further existential graphs rule that can brought into play. This additional rule states that anything that shares the same context with an empty negative context, known as the 'empty clause', as in the above, can be removed. This is because logically the truth of any context which contains the empty clause is always false. The result is (fig 6):

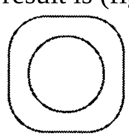


fig 6

The double negations cancel out simply to true. This becomes useful when faced with a problem such as given by the following example (fig 7):

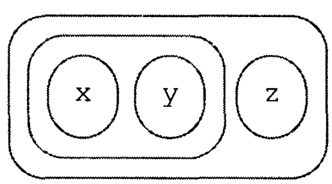


fig 7

This graph also represents $(x \cup y) \Rightarrow z$. From the graph it is evident that x or y will lead to z being asserted by the use of the empty clause rule.

Value* of *Peirce Graphs

The ability to make sense of existential graphs depends on connections with signs and not objects, there are certain spatial effects such as adjacency and inclusion that make the relationships and the structure more transparent. Peirce Graphs offer a clear, graphical, unambiguous way to approach logic teaching. Logic students need only to understand a small number of simple rules and with experience may be able to use the diagrammatic representation to their advantage. No doubt this simplicity is what Peirce would have intended.

However there are a number of problems with this form of representation, none least the degree of complexity that can arise with problems of high order (Polovina and Vile 1997). This form of logic does have its limits, nevertheless it was not intended as a replacement for symbolic logic, merely as a teaching and reasoning tool. Existential graphs seem so good at explicating logical processes that they also lay open these processes to critique in themselves, showing the limits of first and second order 'crisp' logic in support of reasoning and complex, real world decision support (Polovina and Vile 1997)

Some may be critical of existential graphs on the grounds that it is not a usual notation and is therefore not at all useful for intending computer scientists and mathematicians; additionally it becomes very complex in its beta form for first order predicate logic. As such it could be argued that it is only pedagogically valuable in its alpha representation, as a middle step towards more formal (yet constraining?) representations.

Conclusions and the future

We hope that in this short exploratory paper we have been able to illustrate the possible value for logic education of conceptual graphs. Within our experience of teaching logic to first year undergraduate computer scientists we have come upon a number of recurring problems. The majority of these we believe are caused by inability of students to make appropriate sense of the symbolism, and its abstract nature, which in keynote cases (in terms of development in the learning of logic concepts) refuses to correspond with intuition. In other areas of mathematics education this phenomenon has been, and continues to be, examined in detail (for example see Pimm (1996)). Conclusions indicate the possible presence in individuals of *cognitive gaps* (Herscovics and Linchevski 1994) which divide senseful symbol use from incomprehensibility, and notions of reification of signs so that they become treated and manipulated as if they were objects. We want to suggest that problems with symbols and symbolic representations arise from their inherent opacity, and we would argue that initial introductions to mathematical subjects be as concrete as possible. Existential graphs are a good example of a less semiotically demanding representation for complex linguistic and logical structures.

We hypothesise that Peirce logic is a better first approach for logic teaching that any other, and should replace the truth table representation which is the common first port of call. We intend to test this hypothesis with a comparative study during the next academic year and would be happy to report our results at this conference. If our hypothesis proves true then we will develop further dynamic teaching resources and use the evidence to institute a change in our own institution, which may possibly be followed by others.

Peirce "would be the very first to insist that logic cannot be learnt from logic books or logic lectures" (Houser and Klosell992) and he remarks that, although it is useful, a 'mathematical head' is not necessary for the study of logic. To students of our University this would be a great relief. We suggest and hope to establish through research that this is only so if we remove the strain of making sense of formal logic symbolism and replace, or at least precede, it with the more transparent Peircian representation - existential graphs. Peirce had only a short time as a lecturer in logic, not enough to put his ideas to good use in education. We hope to re-open his logic casebook and begin again the clear explication of logic that he began as far back as 1882.

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