Mathematical Support for Engineering and Science Students

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An investigation of the mathematical difficulties experienced by undergraduates starting courses in science and engineering has revealed a range of misconceptions that appears not to have changed greatly over several decades despite changes in school curriculums. Many of these difficulties relate to the understanding of graphs. There is also evidence that the students find the style of learning expected at HE to be greatly different fromwhat they have experienced at school and this too contributes to their problems. A number of reasons for the problems are hypothesised and some solutions are proposed.

Introduction

In the wake of the publication of the Engineering Council report on the changing mathematical background of engineering undergraduates (Sutherland & Pozzi, 1995) we have been investigating the issue within King's. This has involved one education lecturer working with first year undergraduates of physical science and engineering who have been identified as being likely to have difficulties with their mathematics. The initial identification is made using a diagnostic test developed some time ago for such work and modified over several years. The selected students then meet in groups of about a dozen for an extra maths tutorial each week. The content and style of the tutorials are informed by their performance on the diagnostic test and draw on recent developments in mathematics education research and ongoing research taking place in the School of Education, especially CAME (Cognitive Acceleration in Mathematics Education)

Background

Worries about engineering and science students' mathematical competencies are nothing new. Over a quarter of a century ago lecturers in the King's maths department wrote "Many students of science subjects arrive at university with little facility and less interest in mathematics" when describing the reason for setting up a system of mathematics support tutorials (Baker, Crampin, & Nuttall, 1973), and even Sir Ron Dearing in his Review of Qualifications for 16-19 Year aIds commented" ... representations by employers about standards in ... the application of number have been a feature of national life for more than a century. It has always seemed that things were better twenty years ago" (Dearing, 1996)p6. Descriptions of the areas of mathematical weaknesses over the years have had much in common: calculus, especially integration, always appears. Other topics lists regularly include logarithms, trigonometry, complex numbers and algebraic manipulation especially factorisation.

The stability of the criticisms over many decades accompanied by a clear failure to address them, led us to believe that many of the problems were at a much more basic level. Even this is not a new hypothesis, in a paper on the mathematical deficiencies of engineering undergraduates written nearly twenty years ago the authors, having produced a list such as that in the previous paragraph, write

"Our evidence is that the deficiencies are more fundamental than this list would suggest or than students are prepared to admit" (Howarth & Smith, 1980).

Accordingly the diagnostic pre-test addressed mostly topics that traditionally are taught at pre-sixteen level and contained a number of tasks designed for national surveys of school mathematics (e.g. CSMS and APU). The topics covered included simple algebra and number, the laws of indices especially numbers in standard form, and quite a few questions on the understanding of graphs both visually and in terms of $y = rnx + c$.

Findings

Despite the students' having passed mathematics examinations at 18+, there was evidence of a range of fundamental misunderstandings:

(i) A number of the diagnostic test questions were designed to elicit standard misconceptions as displayed by pre-sixteen pupils. Many of the students still displayed exactly such misconceptions.

(ii) Students could often show success in one area but fail when the context or style of the problem was changed. There is evidence of the learning of mathematics as a set of disconnected skills in order to pass examinations.

(iii) There was surprisingly little difference in overall pre-test performance between engineering students with BTEC, physics students with A-level maths, and chemistry students with no more than GCSE maths. However it seemed that the post A-level students were generally slightly better at graph interpretation than the post BTEC students. We suggest that this is because most A-level maths syllabuses require the use of graphical calculators which is not usually the case for BTEC. On the other hand the post BTEC students seem better at number problems. We suggest that this is because they are generally older and have more experience of using number.

(iv) The most significant area of skills and knowledge in terms of predicting success in the university mathematics examinations for the engineers concerned graphs, including the interpretation of shape (especially changes of gradient), verbal interpretation of graphs and the non-calculus algebra of graphs. This connection was extremely strong and further investigation during tutorials and interviews has supported its importance. **In** many ways this is a surprising conclusion because the end-ofsemester examination concerned had apparently virtually no requirement for these skills. We hypothesise that we may have uncovered an underlying cognitive structure that is necessary for success at higher level mathematics. This is being investigated.

(v) Many students are unsure of how to learn mathematics and find the jump from schoollFE to HE to be very great. In a large university the opportunity for individual or small group tuition is limited. King's uses the standard style of large formal lectures backed up by smaller examples classes. However these smaller classes may have up to 30 students in them. For many students, learning is seen as a passive activity and this style of teaching tends to reinforce that view. When asked (in the support tutorials) to talk about what they are doing or to work in pairs and small groups they find it very uncomfortable. There is little evidence of ability or willingness to reflect on their own learning so in the wake of the success of such techniques in the CAME (cognitive acceleration in mathematics education) project, much time is spent helping the students to develop such metacognitive skills.

Discussion

Perhaps the most remarkable finding is the stability of the problem areas despite massive changes in the mathematics curriculum at school level. Twenty years ago school mathematics syllabuses were designed as feeders for university mathematics. With the introduction of GCSE, the National Curriculum and modular A-levels this is clearly no longer true. The mathematics curriculum up to GCSE is very much broader than in the past and is designed generally for purposes other than preparation for HE maths. Such changes, accompanied by the much wider recruitment base of undergraduates should lead to the expectation of a very different range of mathematical problems to that encountered twenty and more years ago. The fact that this seems not to be the case is of considerable interest.

This is not to say that there haven't been *any* changes. Comparison between the test scores of the students entering in 1995 and those of students sitting the test in 1983 show that there have indeed been a number of changes. However subsequent observation and discussion with students tends to indicate that the changes are in the areas of skills rather than underlying concepts. In other words the widening of the school maths curriculum has given students less opportunity to practice mathematical skills (or rather, those mathematical skills required for the HE maths courses under discussion.) because of the range of mathematical study required. For instance the National Curriculum for mathematics includes a considerable amount of statistics of the kind that was only met on undergraduate courses twenty years ago. At the same time this increase in content may have left less time for teaching for conceptual understanding.

Another factor is that of teaching to the test. Teachers have always done this to some extent and it is unavoidable. However the introducing of criteria for GCSE and the National Curriculum accompanied by a tightening up of examination rules makes this easier. In addition because of the introduction of teacher assessment and coursework, many teachers are more aware of assessment theory than in the past. This is producing cohorts of students who are good at passing certain styles of examination regardless of content. See (Gill, 1994) for evidence of this.

There is also the issue of what service mathematics is for. It is quite clear that the students see no relationship between their mathematics courses and their main subject studies. An example of this was revealed in a discussion with a group of physicists at the end of their first term at university who were asked how their studies in mathematics related to their studies in physics both at A-level and at university. They were unanimous that there was no apparent relationship between the two subjects at *either* level. While they agreed that the maths course they study at university would be impossible without A-level maths, they made no other connection. Indeed the most remarkable quote was " ... we haven't used maths in Physics much", a comment which raises questions about what they think mathematics actually is. Further probing revealed that they didn't see the quantitative and algebraic content of physics as being mathematics.

This situation is exacerbated by the messages, implicit and explicit, that the students receive from their lecturers. It is not an exaggeration to say that they are caught in the crossfire of a three way battle for the mathematics curriculum both within the institution and on the wider scale. The views of mathematics held by the mathematics establishment, the mathematics *education* establishment, and the users (science and engineering departments), are frequently mutually exclusive and many of the lecturers concerned are not reticent about expressing these views in public and to their students.

Where now?

The first thing to realise is that rapid wholesale changes in the maths curriculum at school level are simply not going to happen for political reasons, and anyway this investigation is revealing that many of the problems are curriculum proof. However there are some promising avenues. The cognitive acceleration schemes, CAME and CASE alluded to earlier (Adey & Shayer, 1993) have had considerable success in raising cognitive levels *and* examination success in GCSE mathematics and science. Indeed industry (BP) has put funds into the area by sponsoring the training materials for CASE. One of the reasons for the success of these schemes is that they require considerable professional development in the teachers concerned over an extended period (at least two years). Unfortunately that is very expensive.

To reduce the amount of teaching to the test a wider range of assessment techniques need to be introduced at all levels. The frighteningly poor reliability (using the term in its technical sense) of standard written examinations needs to be more widely appreciated. It is perhaps significant that the previous government's assessment agency refused to carry out reliability studies on its National Curriculum tests. One way forward would be to raise the status of Teacher Assessment of the National Curriculum. However this too would require more professional development for teachers with its accompanying funding implications. The scheme of verifiers used in GNVQ would make a good starting model for this.

There have always been problems of discontinuity between different phases of education. The introduction of the National Curriculum improved the primary/secondary link, however there continue to be claims that the shift from GCSE to A-level is too great. In fact research carried out for SCAA (Brown, 1995) indicates that this is not such a great problem as had been thought. There has been little or no recent research into the shift from school to vocational courses, however some work done twenty years ago with vocational students (Furneaux & Rees, 1978) indicates similar problems to those found in the present investigation. Where there clearly is a problem is between collegeNIth form and HE. Many school teachers are unaware of what happens at university and there is no doubt that there is considerable lack of knowledge among HE lecturers about what is happening at lower levels. This may partly be due to lack of interest and head-in-the-sandism, but is certainly exacerbated by the scale and rapidity of the changes at school level at a time when HE itself is in upheaval. Some recently published research into the problem of mathematics for engineering students (Crowther, Thompson, & Cullingford, 1997) identifies this lack of continuity and information *asthe* major factor at play.

The basic content of HE ancillary maths courses has not changed in a generation but incoming students' knowledge has. It has been suggested (Ireson, 1996) that the situation could be improved by considering what the students entering HE *comewith* from their earlier studies. The example of their knowledge of statistics has already been quoted but it is little realised just how IT literate our current students are. Students now entering HE are among the first to have followed the National Curriculum for IT and in many cases they are well ahead of their lecturers. Spreadsheet mathematics has entered most fields and all the new students have experience of it. They find it very odd to be told that the only IT used in the HE maths courses is the standard scientific calculator, not even graphical calculators being permitted in many university departments.

Finally

The stability of incoming undergraduates' misunderstandings over many decades of change has a number of implications, not least that the evidence of educational research is not being taken on board by teachers and textbook writers and perhaps we in the BSRLM should spend more time considering how to extend our influence. However at this stage the biggest problem revealed by this investigation seems to be the lack of cross phase dialogue between school and HE and perhaps this is where to start.

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