Ontogeny, Phylogeny and Evolutionary Epistemology 1 Leo
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Abstract

Reference to the historical development of mathematics has been widely used in discussions of the problems students have in learning mathematics and the devlopment of concepts in the individual. This paper examines the basis for these claims in the interpretations of the history of mathematics that are current in the literature, the universality of the supposed concepts, and the use of the "principle of parallelism " where indivudual development is claimed to mirror the historical development of the subject matter. The conclusion is that the majority of these claims are basically unsound, and that what we learn from the history of mathematics is that the richness, complexity and variety of human endeavour in this field is much greater than hitherto supposed. Consequences are drawn which challenge some of the fundamental tenets of Piagetian epistemology.

1. Introduction

Doubts have been raised (Damerow 1988, Hoyrup 1985, 1994, Rogers 1996,) about the links that have been claimed to exist between ontogeny and phylogeny. This paper discusses the relationship between the ontogenetic development of the individual and the historical development of mathematics found in Piaget (1970), Piaget & Garcia (1983,1989) Bachelard (1993) and Sfard (1995). The assumptions on which these psychological models of human development are based are at such a general theoretical level that their claim to validity goes far beyond what can be verified or falsified by investigations of ontogenetic processes in developmental psychology. Piaget's appeal to the connections between the history and the development of mathematical ideas is extremely general and tends to interpret not only history of mathematics but also the structures of mental operations in terms of modem concepts (Piaget 1970, Piaget & Garcia 1989). I suggest that such generalisation is unjustified, and that contemporary evidence shows that the historical situation is much more complex than at first assumed. Another problem is the claim to universality of application. It has been assumed that Piaget's model of the development of mathematical concepts applied at all times, and everywhere. Serious criticisms of this assertion have shown that differences iil cultural contexts give rise to what are essentially different cognitive structures when it comes to handling number, magnitude and space (Gay & Cole 1967, Hunting 1987).

2. Parallelism in Mental Development and History of Science

In the early days of embryology it became established that animal (and human) embryos developed through a series of stages which observers claimed were found in the 'lower' animals. These ideas were known as Recapitulation Theory. This theory was extended to human children growing up, who were compared to 'savages' and where the 'primitive races' around us were seen as earlier stages of Western civilisation. This theory promised to reveal not only the animal ancestry of man but also the origins of his mental, social and ethical development, and it was taken up in many fields as a • scientific ' basis for value judgments, preserving the status quo and pathologising groups of individuals. Sometimes called the "Biogenetic Law", this principle was adopted by a number of mathematics educators at the end of the nineteenth and in the early twentieth century (Cajori 1896, D.E. Smith 1900, Klein 1908, Poincare 1908 and Branford 19(8) and has persisted in some form or other ever since. Latterly, the strong form of recapitulation has been replaced by the weakerform of 'parallelism'. Branford (1908) gives an extensive description of the "Biogenetic Law" and relates how there was a "parallel" between the stages of learning mathematics and its actual development in history. Part II (1908 chapters XV to XXIV) consists of the evidence from observations of pupils learning which, using the "principle of parallelism" produces the justification for the application of the Biogenetic Law. Superficially, this "parallelism" is obvious. However, it is contrived by a selective and limited interpretation of incidents. Another idea which has a historical basis is the 'epistemological obstacle'. This concept is attributed to Bachelard (1993). The epistemological obstacle forms a key part of his theory of the development of scientific concepts where he makes a parallel between a psychological theory of individual mental development and a genetic theory of the development of concepts in the history of science. Bachelard proposed a "law of three states" of individual mental development proceeding from concrete to abstract; parallel to this there are three stages to scientific investigation from a naive interpretation to the formation of abstract theories (1993 pp.819) These broad descriptions are used to draw parallels between the ontogenetic development of the individual mind and the phylogenetic global development of science. In order to explain the parallel processes of mental and scientific development, Bachelard postulates the following framework: (a) epistemological breaks characterise the way in which 'true' scientific knowledge contradicts our 'common sense' experiences; (b) epistemological obstacles are concepts, methods or customs that prevent an epistemological break. (c) An epistemological profile is an analysis of a given individual's understanding of a scientific concept where the profIle carries a record of the epistemological obstacles hindering the 'scientific thought' of an individual. (d)Finally, an epistemological act corresponds to a scientific breakthrough that can introduce a clear advance over the past, or the preservation of past science through a reformulation of old ideas .

3. Intemalist - Inductive History of Science and Mathematics

The above approaches see the history of science and mathematics as a story of great men and their accomplishments which rejects old, inappropriate and 'wrong' ideas and fashions new concepts which become ever more abstract and general in their application. The situations which generate the obstacles are our naive interpretations of the world which need to be theoretically restructured before we can improve our explanations. This intemalist view neglects the important social contexts in which science and mathematics arise, and the possibility of alternative interpretations. It begs questions about the continuity of records by its assumptions that our present state of knowledge about the past is complete, and it also poses problems about the biased interpretation of data. Viewing past situations from a contemporary point of view suffers from the danger that the interpretation will be in terms of concepts held by the observer, and not in terms of the concepts contained in the situation being investigated. Finally, history of science and mathematics is neither science nor mathematics; the claims that scientific theory and mathematics are in some sense objective and that the progress of science is value free and therefore represents the 'progression of the scientific spirit' are misguided.

4. Social History of Mathematics

The social history of mathematics recognises the contexts of mathematics and mathematicians in a particular period, their ways of communication, their implicit aims, the ways they justified their activities, etc. It recognises mathematics as a cultural activity consisting of interactions between people, mathematicians, or their intermediaries. We have to decide how these social contacts should be valued and what they contribute to the explanation of the development of mathematical practices and theories. This will depend on the personal point of view of the historian, the context of the time, and the purpose of the explanation. In this case, the identification of any epistemological obstacle is located in a social complex of many dimensions. If the epistemological obstacle is seen as a universal problem, then its nature has to be justified for all cases, not just the most spectacular examples recorded in versions of the history of the sciences. However, locating mathematics, and its history in the social domain is immediately problematic for its objectification as understood in the traditional sense. A consequence of this location is that *there is no universal truth, and so there are no universal epistemological obstacles.* Likewise, there is no reason why the epistemological profiles of individuals studying the same problem should be the same. In fact, it becomes impossible to distinguish between epistemological obstacles and the 'obstacles' created by cultural contexts.

5 . *The Basis of Piagetian Theory*

Throughout his work, Piaget advocates forms of "Parallelism" (1967, 1970, 1989) and draws on comparisons between the child's acquisition of logico-mathematical knowledge and the history of Western science. He compares the thinking of 'primitives' with children, suggesting they both have insufficient experience of elementary actions which support the development of concrete operations and reconstructs the history of arithmetic as a process of "becoming aware" of the operations already present in the cognitive structure of the concept of number, a process based on experience. Only when *conservation of quantity* is achieved is it possible for 'more' and 'less' to be expressed by numbers, and in the process of ontogenetic development this structure is achieved as the result of reflective activity on dealing with objects. Piaget interprets these actions as resulting from an internal process of the development of biologically predetermined possibilities interacting wit~ the environment, and so the basic structures of cognition are a special (and particularly human) form of biological coordination of behaviour (Piaget 1967). Epigenetic explanation of the development of concepts is only applicable in the cases where the phenomenon applies to all, but this has been challenged by Bruner et al (1966) who claim that *representations* have a considerable role in the development of conceptual structures. For Piaget's epigenetic conception of the origins of number to hold, we have to ask if the number concept can be described as a universal cognitive structure already present in the minds of human beings as a result of ontogenetic processes of development before corresponding representations appeared in the history of numerical concepts. If this is so we ought to be able to identify, among the widely different developments in the history of arithmetic, a psychological process which corresponds to the synchronous emergence of logical and numerical operations in ontogeny. However, if representations have a significant role in the development of cognitive structures, the consequences are different. Since representations are culture-dependent, the relations between ontogeny and cultural evolution are completely changed. If the ontogeny of cognitive structures is prescribed by different representations, then these representations could exhibit very different stages of development from those currently accepted.

6. Historical Evidence and the Development of the Number Concept

There is great cultural diversity in the different representations of numbers and comprehensive processes of historical development have been described which proceed from the first number representations to the processes of modem arithmetic. Consideration of the discoveries of very early arithmetical techniques (c. 3,OOOBC and before) represent a link between the proto-mathematical techniques of pre-literate cultures and the abstract number concept. Arithmetical techniques using clay tokens which developed in connection with the invention of writing show that in cultural evolution there was *no simultaneous appearance of all the structural elements of the number concept that Piaget claimed was universal.* If different representations suggest alternative possibilities, then this allows different assumptions about the cognitive structures that form the basis of the arithmetical techniques of a particular culture. Interpretation of evidence available suggests that we can ascertain the state of cognitive development in the early stages of the evolution of number concepts which show a distinctly non-uniform variety of concepts in a localised social and temporal context. At present thirteen different numerical systems have been discovered (Damerow 1988, Nissun et.al. 1993) which were all used about the same time (c.5,OOO Be) for a variety of different purposes. Each system was used to represent calculations concerning specific sets of objects but *there is no general number system applicable to all of these cases and no apparent logical reason why they should be organised in such disparate ways. There is also considerable ambiguity in all of these systems.* The signs also have a qualitative meaning; they describe the products, the objects, and the special measurements and the relations between them required in each case.

7. *Conclusions*

The representation of objects by clay tokens shows the same connections with particular concrete contexts of application and action as does the arithmetic of "primitive" cultures particularly through the simultaneous representation of quality and quantity; and means that quantities are represented by one-to-one correspondence and the additive operations are carried out entirely enactively by the quantitative repetition of the transaction using the tokens. Development of such techniques presupposes only the ability to give the tokens a meaning (for example where each token symbolises an individual animal, object or quantity of goods), and manipulating them only presupposes the ability to anticipate the results of such actions. Consequently there is *no reason to assume that at the same time all the other structural elements were abstracted which Piaget claims are conceived as universals of the number concept.* It is clearly not possible to draw general results from a single example, but this does demonstrate that the study of the development of early civilisations offers possibilities for clarifying basic problems of a theory of cognitive structures which relates ontogeny and historical development. It is important to realise that these observations are not advocating a revision of 'recapitulation theory'; this is an attempt to use contemporary interpretations of historical evidence to show that in contrast to the neo - Piagetian view, emerging concepts are diverse, disparate, culturally located and unlikey to suggest that there was in any sense a 'universal' ontogenetic development. In view of the discussion presented here, the consequences for much of our current approach to some of the fundamental ideas of cognitive development which attempt to link past mathematics with the present conditions of learning may have to be revised.

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