The Object/Process Duality for Low Attaining Pupils in the Learning of Mathematics

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Abstract
Various studies have indicated the difficulty that low attaining pupils have with the object/process nature of mathematical entities. It is further suggested that their "process focus" hinders mathematical development. Using a case study approach I have bee" working with 13/14 year old pupils and probing the way that they operate in both a number environment and a Logo environment. For both of these environments I will share/discuss some pupil episodes and draw some conclusions about the way in which they work with the object/process duality.

1.0 Introduction

Over the last two years I have been working with a group of pupils who are low attaining in mathematics. All the pupils, whose ages were 12/13 years, were judged to be working at level 2/3 of the National Cuniculum for England[l]. Case studies were developed for 8 pupils and data was collected from a variety of sources: individual interviews focusing on a variety of mainly numerical questions, paired work in Logo involving problem-solving tasks, individual projects, Logo-based work undertaken away from the computer and which involved the resolution of conflicts, individual interviews and written work based on Logo tasks. The evidence from these sources provided rich data, the analysis of which enabled me to focus on how tbese pupils give meaning to the mathematics embedded within the tasks on which they work. In particular I focused on the nature of the object/process duality within their thinking as they worked on the tasks.

2.0 Background

Underpinning the work is a belief that all pupils construct some meaning for the work that they undertake and that the way in which they construct the meaning helps them to develop their knowledge and understanding of mathematics. In considering the nature of constructivism Cobb et al (1992 p.2-33) consider it to be an alternative to a *"representational view of mind in mathematics education"*. This sees learning as a process in which students work on their internal representations so that they mirror the external representations to whiclu the student is exposed.

IThe assessment of mathematics in the National Curriculum is assessed on a 10 level scale for pupils aged 5-16 years. Level 2/3 should be attained by pupils of7 years of age.

The difficulty is that the communicator of the external representation already has an internal representation to which he or she relates the external representation. The student only has the external representation through which meaning is to be developed. A representational view locates the source of meaning outside the student whereas the constructivist approach locates the meaning within the student and hislher environment. The representational view appears to set up a dualistic conflict within the student between "in head" or internal representations and external ones which are located in the environment whereas the constructivist approach would seek to start with the internal representations and seek ways in which they can be externalised and then modified.

In constructing their mathematical knowledge there are two aspects of pupils' work that need to be considered - the objects with which they work and the processes which they perform. In mathematics we work with a variety of objects and in each area of mathematics" knowledge is developed by performing processes on the objects being considered. The position taken with this work is that pupil's perception of the mathematical entities with which they work will exercise a major influence on the progress that the learner is able to make. Further, the way in which they operate on these entities is affected by this perception.

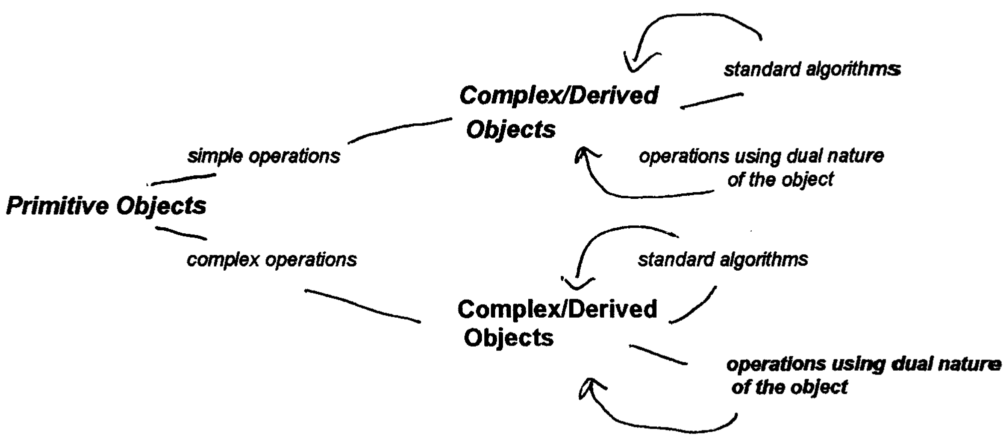
The duality of mathematical entities is developed by Tall (1996) where he discusses "ways of seeing" mathematical entities. He considers mathematical entities in terms of objects and processes and develops the idea of a procept which he defines as:

> *a combined mental object consisting of a process, a concept produced by that process and a symbol that may be used to produce either.*

This leads to a discussion of ways of thinking - procedural thinking where the focus is on processes and proceptual thinking where the stages in symbolic manipulation are compressed and Tall contends that the symbols are viewed as objects that can be decomposed and recomposed in flexible ways. This leads Tall to the idea of a proceptual divide that could distinguish between those who progress well in Mathematics and those who struggle with Mathematics.

3.0 Results

In the tasks undertaken by the pupils the meaning they were able to give to the work was directly related to the objects with which they worked and the processes they were able to perform on the objects. Thus in order to analyse the work of the pupils, Tall's ideas were developed to include the notion of primitive objects, operations on primitive objects which give rise to complex objects, and then operations on the complex objects. The evidence from the case studies mgWights the object/process duality and the difficulty that these low attaining pupils have in thinking about and using the duality effectively. In working in both Number work and Logo work the observed phenomena are discussed and can be represented diagrammatically as:



In each area observed, the pupils worked with a number of *primitive objects.* With these primitive objects they did not need to consider the underlying process aspect of the object - how it was created - and used them as entities on which operations were performed. They were entities which they did not need to break down any further. On these primitive objects *simple operations,* such as adding, subtracting or sequencing, were performed in order to create new and more complex objects with which they could work further. For the pupils in the study there were a small range of numbers which could be characterised as primitive objects - for some, numbers less than 5, for others, numbers up to 10. On these primitive objects they were able to perform simple operations - addition and subtraction but generally not multiplication or division. If they were faced with larger numbers they would, almost invariably, try to perform a standard pencil and paper algorithm on the number i.e. treat it as an object on which an operation had to be performed. This is illustrated by one representative example from one of the pupils - Carol - in her responses to some of the numerical calculations presented as word problems, or as direct calculations both horizontally and vertically. In all calculations her work was as below:

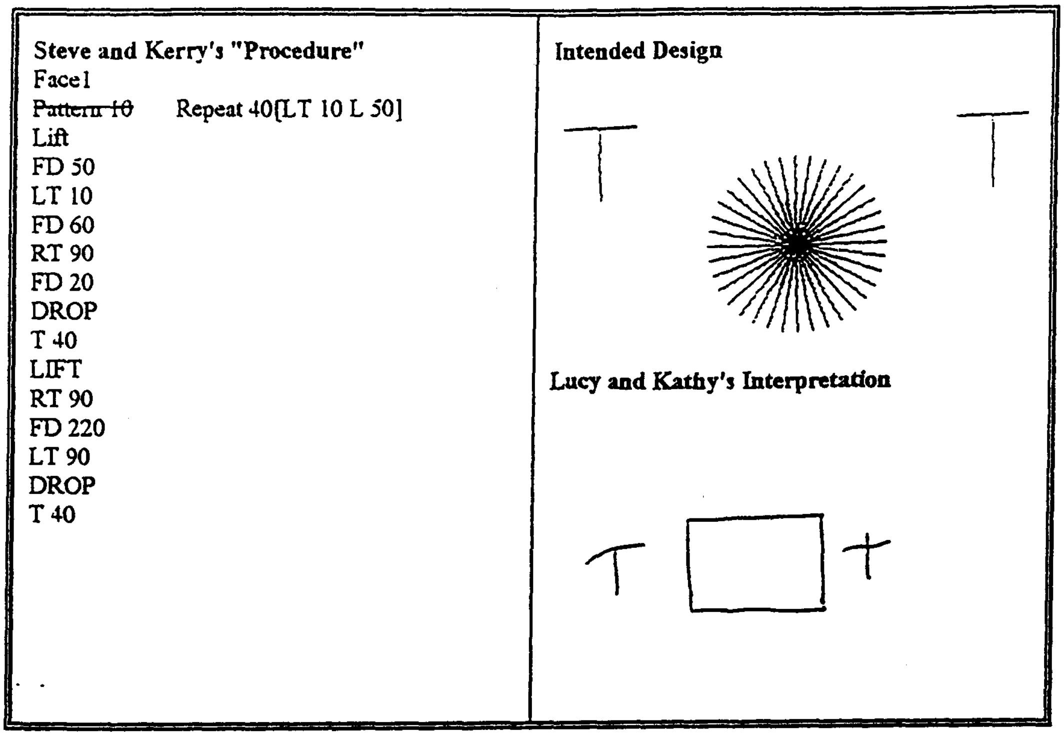
49	26	23
22+	15+	48+
611	311	611

Her description of what she did for 49+22 was:

CC. *I put the two numbers down underneath each other.*
9 add 2 is 11
4 add 2 is 6
That's six hundred and eleven

In each case Carol gave a similar description and wrote the sum in a column. She then performed a remembered process on the given numbers. Her consideration of the correctness of the answer was then judged against her perception of the accuracy of the process and not on the basis of the objects with which she was working. Further evidence of this was shown in the work of the other pupils in addition, subtraction and multiplication work.

In the computer-based Logo tasks the pupils also separated objects and processes inritially. This is encouraged by the nature of the language, in which primitive commands are given and these need to be sequenced in order for a design to be produced. These primitive commands were used to build complex objects in the form of named procedures. In order to be used or manipulated effectively these complex objects (named procedures) needed to be viewed both as objects in their own right and also as the result of a process. They were often used in such a way that the position and orientation of the turtle was not taken into account. In other words the process embedded within the procedure was not considered. The pupils' strategy for overcoming difficulties in Logo was to revert to the use of long sequences of primitive commands. This created more work for the pupils because they found it difficult to think with these new complex objects. As indicated earlier a variety of tasks were undertaken in the Logo environment. In one of these one pair of pupils produced a procedure which another pair had to interpret. They then came together to discuss the accuracy of their work. One of the results is shown below together with a transcript of the resulting conflict resolution dialogue.



The following is a discussion of the way in which Kathy and Lucy discussed how they interpreted Kerry and Steve's procedure - listed above

m.	*Can you explain what you did?*
Lucy.	*You draw a square.*
m.	*Why did you draw a square?*
Lucy	*Because that's the way we thought it was, like that there.*
TH	*What made you think that was a square?*
Lucy	*I don't know . .Just presumed*
TH	*OK. Is there a reason why you presumed that was a square?*
Lucy	*Because of what it is. REPEATed I don't know.*

Here Lucy and Kathy focus on a particular clue (in this case REPEAT) and draw conclusions about this without probing any deeper in order to see what the REPEAT means in this particular circumstance. They have focused on a result of using the REPEAT command rather than focus on the nature of the REPEAT command itself

They then explained how they went through the sequence listed ...

Lucy ... we went forward 50. left 10, and forward 50, right 90 forward 20 drop. The1l draw the T 0J O.

Kathy I didn't understand We didn't know the turtle did we?

Lucy No because they didn't have it listed did they?

In verbalising the way in which they worked through the sequence they have identified detail which they had partially ignored. Firstly Kathy refers to the orientation of the turtle which they had not considered previously, and secondly Lucy refers to the fact that they drew the letter T without knowing the process by which it was originally drawn. Both these - process and orientation - are important aspects of Logo which need to be taken into account when working in this environment. Further Lucy has recognised the process and object nature of the procedure T and both aspects need to be taken into account in order to use the procedure accurately.

4.0 Implications for Teaching

For the pupils in the study the initial interview indicated that they had a process-based approach to their mathematics and this was a consequence, at least in part, of the style of teaching which they had experienced. In other words there were didactical obstacles that needed to be considered in focusing on the mathematical development of the pupils in the study. These would seem to have originated from the style of presentation of mathematical experience which they have received during their formal education. In the initial interview it was seen that the pupils in the study had an almost one-dimensional process approach to the numerical work that they undertook .. They rarely considered the objects with which they were working and thus often undertook more work than was necessary. The study suggests that if the teaching emphasises the process side of mathematics at the expense of the development ofa range of complex objects with which they can work, then for these pupils it can lead to an over-reliance on formal methods of approach, a lack of flexibility and often time-consuming short-step methods of approach. These are didactical issues that need to be addressed. Further a process emphasis in the teaching of mathematics can lead to an error analysis approach to the work of these pupils - that is an approach to teaching which seeks to identifY and correct errors in the conduct of a process without considering the reasons why a pupil may be making these errors. An alternative would be to develop an approach which helps the pupils to build a range of objects with which they can work and to which they are able to attach meaning. This would help the pupils to develop a parallel interpretation of mathematical entities as both objects and processes - an approach which would help the pupils to think proceptually and hence more efficiently.

Selected Bibliography:

Cobb P. et al. (1992) A Constructivist Alternative to the representational view of mind in Mathematics Education in Journal for Research in Mathematics Education Vol 23 No.1. pp. 2-33.
Cobb P et al. (1995)Leaming Through Problem-solving: A Constructivist Approach to Second Grade Mathematics in Murphy P (ed) Subiect Learning in the Primary Curriculum" Open University Press, Milton Keynes.
Glaserfeld Evon (1987) Learning as a Constructive Activity in Problems of Representation in the teaching and learning of Mathematics (ed) Janvier C. LEA publishers, Hillsdale
Glaserfeld Evon (1995) Radical Constructivism. Falmer, London
Gray E (1993) The Role of Symbolism in Arithmetic and Algebra in ESRC Seminar Group Papers. unpublished London Institute
Gray E & *Tall D* (1993) Duality, Ambiguity and Flexibility: A Perceptual View of Simple Arithmetic, in Journal for Research in Mathematics Education. Vol. 25 No.2 pp. 116-140. Harries A V. (1997) Developing Algebra Conce.pts with Low Attainers working in a Loa:o Environment unpublished PbD thesis, Bristol University
Tall D. (1996) Can all Children Climb the Same Curriculum Ladder - Plenary Lecture in the Proceedings of the Eighth International Conference on Mathematics Education, Spain