DYNAMIC GEOMETRY IN THE CLASSROOM - A SOCIO-CULTURAL ANALYSIS.

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This paper sets out to describe a background in socio-cultural theory and ideas of conviction and proof which can be used both to analyse and inform the classroom use of dynamic geometry. The particular context is the use of the TI92 hand-held computer with 12 *year old children in comprehensive schools in the UK Conclusions are drawn about the dialectic in which conviction/proof is reached, and the role of mediation by teacher and software in that dialectic.*

Socio-cultural background

The works ofL. S. Vygotsky have continued to influence Western educational thinking since translations ofVygotsky's works became available in the sixties (Vygotsky, 1962, 1978, Luria, 1979, Cole, 1979). For wide-ranging reviews of the field see Wertsch and Tulviste (1992), Newman and Holzman (1993), Confrey (1995), and Daniels (1996).

Two main aspects ofVygotsky's thinking have influenced later work. The first is the principle that "Social relations or relations among people genetically underlie all higher functions and their relationships" (Vygotsky, 1981a: 163). Mediation is the vector for learning by tools such as . psychological devices (mnemonics etc.) (Vygotsky, 1981b p137), speech (internal, egocentric, and . social) (Kozulin, 1986), the teacher (Jones, 1997), the group (Lave and Wenger, 1991, Hudson, 1998) the class (Hedegaard, 1990) and the screen (Hoyles et al, 1995).

Secondly, the site for mediation is the Zone of Proximal Development (ZPD). Vygotsky (1962) originally defined the ZPD in terms of development and the term 'scaffolding' was introduced by Bruner (1985). Other contemporary definitions included those based in assessment (Vygotsky, 1956 pp 446-448), and those founded on the distinction between everyday and scientific concepts (Kozulin, 1987 xxxiv). More recent definitions have related the ZPD to activity theory (Engestrom, 1987 p174) and to the class and teacher as a whole (Hedegaard 1990). For a discussion of the move to a broader cultural definition of the ZPD see Lave and Wenger (1991, p 48).

The purpose of the present study is to examine the way mediation by tools (among them the screen, the teacher and the group) takes place in th~ context of dynamic geometry in the classroom, looking particularly at the interaction between spontaneous and scientific concepts.

Intuition, proof and meaning-making

Davis (1993) argues for the interpretation of'1he word 'theorem' in a sense that is wide enough to include the visual aspects of mathematical intuition and reasoning". Polya (1954, p vi) points out that a theorem, an example of demonstrative reasoning, usually has it~ genesis in plausible reasoning. " You have to guess a mathematical theorem before you prove it; you have to guess the idea of a proof before you carry through the details."

Mason (1991) refers to 'inner screens' and makes a plea for visualisation, for 'saying what you see' and for an awareness of 'the fact that there are facts', He argues for a form of proof based on a large number of examples as afforded by dynamic geometry.

Fischbein (1982) identifies three forms of conviction: formal, arising from argument, empirical arising from a number of practical findings, and an intuitive intrinsic conviction which he calls 'cognitive belief, It is suggested that the dynamic geometry environment can complement these ideas with discussion and argument based on screen images, with dragging to provide empirical proof and with children's intuitive visualisations which are triggered by the screen. There is a dialectic of proof (or conviction, which at this level (V8) is seen as almost interchangeable with proof) which moves between these areas, which can be related to the ideas of spontaneous and scientific concepts offered from socio-cultural theory. There is a place in this dialectic for the way children deal with necessary and sufficient conditions in mathematical argument.

Hand-held Dynamic Geometry in the classroom.

Dynamic geometry provides an environment in which geometric meaning-making can be mediated by the screen, and particularly by the dynamic image; a socio-cultural analysis is presented here. (This study also looked at how the hand-held nature of the T I 92 allows the development of a 'whole class ZPD' (Hedegaard, 1990) and this will be the subject ofa further paper.)

The drag function and the idea of a construction invariant under drag are central to the use of the software to mediate children's learning. In introducing the use ofCabri, the classroom material was directed at making a distinction between drawing and construction, and at seeking an understanding of concepts such as that of using a circle to preserve length (Healy et al, 1994a, 1994b). Hoyles et al (1995) consider the interdependence of construction and proof and the replacing of proof by construction in a dynamic geometry environment. Healyet al (1994b) refer to the importance of on-screen scaffolding. Pupil fluency with the machines was an objective of early work (Goldstein et aI, 1996).

Data analysis

In one session a group of Y8 pupils was asked to draw a square.

D *I know* **!** *You could do it two triangles, two right angled triangles next to each other and merge them, then it'd be a proper square.*

R *I think I've got a peifect square here.*

J *See, J've just figured out mine's not right, cos one of my lines is* 1.91 *cm and the other is 2.03 cm*

J *There's also area,' you can do the angle and see if the angle's a right angle, as well.*

R *Well you can tell if it's a right angle.*

J *Yeah but you can 'tfor definite*

There is much toing and froing here between inner and electronic screen, between ideas of drawing and construction, between scientific and spontaneous concepts:, and between necessary and sufficient conditions. These children are operating in a dialectic of proof, informed by their ideas and the insights available to them on the screen.

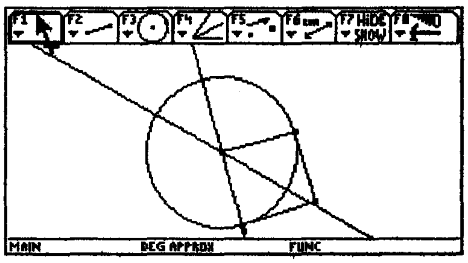

Figure 1 (D)

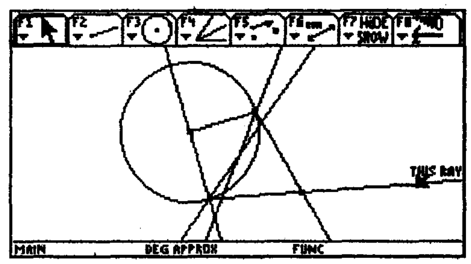

Figure 2 (R)

In later work D (Figure 1) and R (Figure 2) both used a circle, a radius and a perpendicular through the centre as a starting point. There the rigout ended. D had drawn two segments, again by eye, to complete his square. Dragging showed him that the point was not defined. R had defined a point where he estimated the other comer of the square to be and drawn two rays through that point. He drew two angle bisectors, which coincided originally but separated if he dragged the undefined comer of the square

This conversation took place.

D *I'm trying to do an angular bisector .... cos if the angular bisectors make a right angle in the middle then that'll mean it's a square, but I can't get it to do them.*

JG *How do you know that the angle bisectors will meet in the middle in a right angle?*

D *Well I don't know that they will in a right angle*

R *They will*

D *If it's a proper square then it'll be in a right angle becau.se you'd be chopping the square like diagonally*

R *There'd be four triangles*

D *There'd be like four triangles and they'd all be right-angled triangles*

R *There'd be two* 45° *angles*

(Shown how to draw angle bisector)

I
S

D *Yes!! Now that looks like it's going at a* 45^0 *angle right through.*

That meets in the other corner there, so I think that means it's a square.

Here again there is interchange between different levels of vi sua lis at ion, conviction and mathematical argument. A sufficient definition of a square is eventually arrived at, only to be abandoned for the germ of a new approach.

There followed an attempt to probe understanding of geometrical isometries. This conversation refers to Figure 2.

JG *So this circle is a good starting point isn't it?*

If you have that circle and that ray, how many sizes of square can you draw?

R *just one*

JG *as soon as you've drawn that and that*

R *Once you've drawn the circle then you've got the size*

Pressing the grab key when the cursor is away from the diagram makes the independent points in the diagram flash.

JG *Whatjlashes?*

R *That corner there.* (The centre of the original circle) *Does that mean that's the only corner that can be dragged?*

JG *That's the only* Doint *that can be t{ragged. Tell me what you drew first.*

R *I drew the circle first*

R went on to discover that, having completed the construction, he could grab the circumference of the circle as well as the centre and so alter the size of the square, and alter the orientation of the diagram by dragging the original ray.

Concluding Remarks

Mason (1991) suggests that we learn geometry" ... to gain direct contact with the world of mathematics accessed through the mind" and that we can teach geometry" ....by bringing attention to the power of mental imagery, and extensions of the mental screen on paper and electronicall y . "

This work has demonstrated how hand-held dynamic geometry can be used to consolidate children's meaning-making in the areas of construction and proof, and has given an indication of the way in which visualisation can move from inner screen to paper diagram, to electronic screen and back again, taking with it as it does ideas of proof and conviction, and pointing to opportunities for further mediation on the screen and by the teacher.

Vygotsky wrote about the interplay between everyday and scientific concepts:

"In working its slow way upward, an everyday concept clears a path for the scientific concept in its downward development. It creates a series of structures necessary for the evolution ofa concept's more primitive, elementary aspects, which give it body and vitality. Scientific concepts in turn supply structures for the upward development of the child's spontaneous concepts toward consciousness and deliberate use." (Vygotsky, 1962: 109)

It is proposed that the episodes described here give an insight into how mediation by the teacher and the screen can operate in a ZPD defined as the area in which spontaneous and scientific concepts interact.

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