NUMERACY AND LOW ATTAINERS IN MATHEMATICS

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Abstract

The teaching and learning of Mathematics (and in particular number) has been afocus over a long period of time. This matter has been highlighted in a number of recent reports (Ofsted, 1993,1994, *Burghes* 1996, *and Bierhoff* 1996), *which have suggested that there are a large number of low attainers in Mathematics. During the last year* we *have worked with Key Stage* 2 *teachers as they focus on developing the numerical capability of pupils who they deem to be lowattainers. Much _research has concentrated on the errors that these children make as they perform operations. With these teachers* we *have been working on the meaning that pupils bring with them instead of using an error analysis approach. In this paper* we *discuss the \.my in which pupils work with numbers while performing addition and subtraction calculations.*

1.1 Introduction

The Cockcroft report (1982) and more recent reports and studies express concern about the numerical competence of school pupils at all stages of their education (OFSTED reports: The Teaching and Learning of Number in Primary Schools 1993 & Science and Mathematics in Schools 1994). Further, comparative studies (Burghes, TES 15.3.96; Bierhoff 1996, Laying the Foundations for Numeracy) across a number of countries indicated that British schoolchildren lagged behind their counterparts in other countries as far as performing simple numerical operations were concerned. The OFSTED reports express particular concern for the numerical capability of the lowest attaining 20% of pupils studying at Key Stage 2 and Key Stage 3. The report from the London Mathematical Society (Tackling the mathematics problem 1995) suggests similar concerns.

1.2 Object/Process Duality

In constructing their mathematical knowledge there are two aspects of the pupils' work that needs to be considered - the objects with which they work and the processes which they perform. In Mathematics we work with a variety of objects - in number work these will be numbers, fractions, percentages, etc. Each area of mathematical knowledge is developed by performing processes on the objects being considered. Very early on in their mathematical development there is an emphasis on the processes being performed and the techniques required in order to perform the techniques. The emphasis is on using the correct procedure to perform the process. Where pupils fail to follow this procedure, this focus can lead to an error analysis approach to the learning of mathematics as demonstrated by the work of Rees (1992) and van Lehn (1993). Sequences are studied, errors identified and correction strategies are put in place. This emphasis sees the objects as entities on which processes are to be performed and does not give much emphasis to the fact that the objects are themselves the result of processes. Thus an alternative approach is to focus on the objects with which we want the children to work and to develop an understanding of how the objects are derived.

This leads to a focus on the meaning (Lins 1992) that we give to the objects with which we are working and on how that meaning can help to make the processes that we perform on the objects more efficient.

The position taken with this research is that the pupil's perception of the mathematical objects or entities with which they work will exercise a major influence on the progress that the learner is able to make. Further, the way in which they operate on the objects is affected by this perception. The idea of using objects to create meaning is discussed by Fuson et al (1997) where they talk about objects as "meaning makers".

2.1 The Study

During the Spring and Summer terms we have worked, on an INSET programme, with 15 KS2 teachers who wished to focus on issues related to special needs in the normal mathematics classroom. Some of the work was college-based and other parts were school-based. The study is based on the results of school-based tasks conducted during this time.

The aim of the study was to explore the ways in which pupils; in these teachers' classes, explained their approach to addition and subtraction calculations. We wanted to explore any differences in the way in which the pupils approached their calculations and how this might inform work with low attaining pupils in Mathematics

2.1.1 Methodology

The ftrst task was to analyse the different types of addition and subtraction calculations that the pupils were expected to undertake. The addition categories are illustrated in the diagram below:

Addition	Categories	Addition	Categories
U+U	no crossovers	TU+T	no crossovers
U+U	crossovers	TU+T	crossovers
T+U		TU+TU	no crossovers
T+T	no crossovers	TU+TU	crossover from U only
T+T	crossovers	TU+TU	crossover from T only
TU+U	no crossovers	TU+TU	crossovers from T and U
TU+U	crossovers		

Having undertaken this analysis we now needed to gather information from the pupils. We decided to use the teachers as this would mean that the pupils were in a familiar environment and so would feel less threatened than they would if we had undertaken the tasks ourselves. From their experience of the pupils the teachers identifIed the pupils as high, average or low attainers in Mathematics and were given sets of questions to work on with the pupils. These were accompanied by specifIc instructions about how to conduct the activity. This enabled us to gather information about the three categories of pupil and to make some comparisons.

Addition Task

For this activity, sheets were produced for the pupils for each of the categories identified above. In anyone class only a limited number of categories was available - a maximum of three. The procedure that was followed is indicated below:

- *The pupils worked in pairs.*
- *Each pupil received a sheet with the sums taken from one category.*
- *The pupils looked at the sums and decided how they would approach each one individually.*
- *For each sum one pupil explained the method used to the other pupil.*
- *The pupils agreed how they would explain the method used to the rest of the class and if possible they would make a note of the method used on the RHS of the sheet.*
- *The previous two steps were repeated with the pupil roles reversed.*

Where pupils had difficulty in recording their method the teacher scribed for the pupils. He/she then organised a structured feedback with the class on the work that the pupils had completed. The teacher also indicated on the sheet the perceived potential of the pupils as low, average or high attaining in mathematics.

Subtraction Task A similar procedure was used for subtraction ..

3.1 The Results

3.1.1 Pupil Responses - addition

	low attainers	**average attainers**	**high attainers**
Number of responses	227	388	358
Number of non-responses	38	22	2
% of non-responses	**14%**	**5%**	**0.01%**

This table shows a much higher non-response rate from low attainers. It could be conjectured that reasons for this are related to pupil confidence, and their facility to communicate their thinking. This is something that could be explored further.

From an analysis of the questions completed by the pupils the following strategies were identified

Category	**Strategy Used**	**Category**	**Strategy Used**
1	Starting with the tens digit	7	Instant recall
2	Starting with the units digit	8	Starting with the largest number
3	Rounding	9	Keep one number intact, partition the other, and then add Ts & Us or Us and Ts
4	Counting on in 1s or 10s	10	Use complements of 10
5	Pivoting around 5 or 50	11	Swap units digits
6	Reinterpret as x	12	Used doubles

Some examples of the way in which pupils responded to questions are given below:

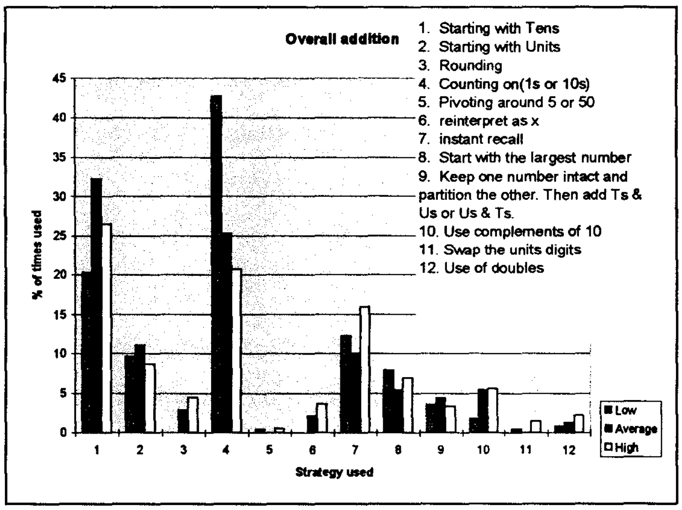
Category G (TU+U with Crossovers)

23+7	I added 7 to the 3 and that made 10. So I added 20 and 10 and that made 30 (Low attainer, aged 10)
55 + 7	I pretended that the 9 was 10 so I added on 10 and took away 1 (High attainer, aged 8)

Category TU + TU with crossovers in U

48 + 23	I took 3 off 20 and added it on to the 48 which made 51 + 20 = 71 (Average attainer, aged 9)

For all categories used we identified the number of times each pupil used a particular strategy. The results were combined to give an overall view of the strategies used by the pupils. These results are illustrated in the graph below:



Comments

From the results of the exercise the following points can be made:

- *The variety of methods used by "high" attaining pupils is greater than that used by other groups.*
- *Low attaining pupils predominantly use a counting method for their calculations.*
- *Concentration on objects led to morejlexible lmYs of working.*
- *There are didactical implications which arise from this analysis which are discussed later in 4.1.*

3.1.2 Pupil Responses - Subtraction

There is only sufficient space here to give a brief summary of subtraction responses.

- *Responses to subtraction categories indicated a high non-response rate from the lowattainers.*
- *The categories and strategies used in most cases were similar to addition. Where there were differences these are identified below.*

Category	Strategy Used	Category	Strategy Used
4	counting on/back = + 1, or +10 at a time	9	Keep one number intact
8	Knowledge of addition facts	11	partition other number and then subtract Ts & Us or Us & T

- *An overall view of the strategies used* lillS *gained in a similar* lillY *as for addition. It* lillS *apparent once again that the lowattainers showed much less flexibility in their choice of method than the higher attainers, and their work* lillS *dominated by limited and time-consuming counting strategies.*

4.1 Preliminary Conclusions 4.1.1

Some Implications for teaching

The results of the study suggest a number of implications for teaching. The process with which the teacher and children are engaged is that of teaching and learning rather than training and being trained. This implies that the thinking processes of the pupils are more relevant than specific overt responses. In this case linguistic communication becomes a process for guiding a student's learning not a process for transferring knowledge. When ihe learner appears to deviate from the teacher's expectations it provides an opportunity for exploring the basis on which the learner gives meaning to the situations in which they are working. This links with the work of Fuson (1997). There is a tendency particularly with low attaining pupils to follow an error analysis route rather than for the teacher to perceive the children's decision as rational ones. However it could be equally important for the teacher to consider the explanations provided by the pupils, since these explanations will reflect the direction in which the pupils are thinking. The role of the teacher will be to explore and broaden the understanding behind the explanations and to help the pupils to adapt their knowledge base.

From the work undertaken it would seem that there are particular strategies that low attainers may need to be taught so that they can make choices about the most appropriate way to approach the calculation. These strategies would involve the pupils in focusing on the objects with which they are working rather than the process which they are performing on the objects. This object focus could lead to a consideration of strategies involving: *complements to 10, rounding, starting with the tens digit, doubling, and the use of multiplication tables.*

Another important issue concerns the nature of the communication between the pupil and the teacher in developing this range of strategies for calculating. The teacher needs to develop a particular kind of mathematical culture in the classroom which encourages flexible ways of working. It is envisaged that a wider range of informal strategies should be used by all pupils in the classroom if they are to be confident and effective mathematicians.

A further didactical consideration is that illustrated in the Kings College Study on effective teachers of numeracy (Askew et aI1997). Here they identify characteristics of three orientations of teachers-connectionist, transmission and discovery. The characteristics of the "connectionist orientated teacher" are given below for two of the aspects considered:

Beliefs about pupils and how they learn to become numerate
• *Pupils become numerate through purposeful interpersonal activity based on interactions with others* • *Pupils learn through being challenged and struggling to overcome difficulties* • *Most pupils are able to become numerate* • *Pupils have strategies for calculating but the teacher has the responsibility for helping them refine their methods.* • *Pupil misunderstandings need to be recognised, made explicit and worked on.*
Beliefs about how best to teach pupils to become numerate
• *Teaching and learning are seen as complementary.* • *Numeracy teaching is based on dialogue between teacher and pupils to explore understandings.* • *Learning about mathematical concepts and the ability to apply these concepts are learned alongside each other.* • *The connections between mathematical ideas need to be acknowledged in teaching.* • *application is best approached through challenges that need to be reasoned about.*

These orientations are clearly desirable in our teachers if they are to be able to respond to the needs of pupils- particularly those who are low attaining in Mathematics. The question then is how are we to develop these orientations in both trainee primary school teachers and practising primary school teachers. This style of approach to teaching and learning requires the teacher to be confident about developing connections across Mathematics and about probing the meaning that pupils give to the numerical tasks that they work on. To do this they need to be given the opportunity to. develop these attributes themselves.

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