

A CONJECTURING ATMOSPHERE IN 11-16 MATHEMATICS CLASSROOMS:
FACTORS AFFECTING ITS CREATION AND MAINTENANCE

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Abstract:

From a standpoint that school mathematics lessons should allow children to function as mathematicians, it is argued that crucial processes are "generalising" and "stressing and ignoring" as participants search for "seeing the general in the particular".¹ This, and a broadly social-constructivist perspective, provided the writer with an image of the classroom within which conjecturing and discourse are not only routine, but essential. My early focus was to explore the dynamics of this social arena. Interviews with 40 or so children suggested that peer pressure was hugely influential. Many of the children interviewed reported that much of what they did and said was based upon the reaction, both actual and predicted, of other children. Other areas to emerge from these interviews were: teacher behaviour; classroom discipline. Almost three years on, I am currently in the process of re-examining the image of the "conjecturing classroom" that I began with.

conjecture *n.* 1. The formation of conclusions from incomplete evidence; guess. 2. the conclusion so formed. ~*vb.* 3. to infer or arrive at (an opinion, conclusion, etc.) from incomplete evidence.
conjectural *adj.* involving or inclined to conjecture.

Before reporting on factors that affect how a conjecturing atmosphere is created, the case for why it is important has to be established.

"The fundamental task of the teacher could be conceived as being concerned with finding activities to offer pupils which allow them to operate as mathematicians and practise on and develop those powers of the mind which are characteristic of the mathematician. ...²

Gattegno argues that mathematics is easier to recognise than define³ and this was a sentiment that (a) I realised (at some point) that I agreed with and (b) found rather unsatisfactory. Gattegno's "solution" was to replace the word mathematics by a process or set of processes "which can be made more tangible". He calls the process *mathematization*. This change of form from noun to verb was a

¹ Mason, I. (1989) Teaching (Pupils to make sense) and Assessing (the sense they make) in *Mathematics Teaching: the state of the art*, P.Ernest (00) Falmer Press pI 55

² Hedger, K. and Kent, D. (1979) "Armes' Theorem or, 'Go on sunshine- guess!'" in *Mathematics Teaching*, 89, p49, AIM

theme explored by Papert⁴ whilst advocating that children should be taught to be mathematicians rather than taught about mathematics.

" .. being a mathematician, like being a poet or a composer or an engineer, means doing rather than knowing or understanding. "

But- what are these processes? What does it mean to be a mathematician?

Throughout his work, Gattegno repeatedly refers to the powers of very young children. There is a suggestion .. often far from subtle .. that the achievements of very young children make the demands of 'ical" school mathematics courses seem trivial. For him, crucial processes are "generalising" and "stressing and ignoring." He has others, but it suits my immediate aim to stress these and ignore others.

In 1874, Felix Klein⁵ referred to the "study of invariant properties with respect to a group of transformations." Although in this context he was specifically involved with geometry, in 1923 he commented that he did not "conceive of the word geometry onesidedly as the subject of objects in space but rather as a way of thinking that can be applied with profit in all domains of mathematics." John Mason ⁶ refers to "seeing the general in the particular, the sameness in different events" as "one of the root processes in mathematics, and probably in every discipline".

Over a period of time, I became clearer what, for me, were key mathematical processes. At this stage, agreement with the reader is neither sought nor is it important. Of importance is the frame of mind this writer / researcher / teacher / head of department was in regarding the nature of the subject, for this provides an essential backdrop when considering offerings.

My working definition was that mathematics:

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| <ul style="list-style-type: none">● needs to be enacted rather than received● involves actions as well as words● brings about an internal change in the participant● involves the following "processes":<ul style="list-style-type: none">● searching for pattern and structure● simplification● reasoning● specialising● generalising● stressing and ignoring● making and testing hypotheses | <ul style="list-style-type: none">● explanation● proof and refutation● classification● representation● interpretation
● ..and that these process are applied when searching for invariants in a transformed situation● ..is also a body of knowledge that can be re-explored by focusing on these processes |
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³ Gattegno, C. (1987) *The Science of Education Part 2b: awareness ofmathematization*. Educational Solutions

⁴ Papert, S. (1972) 'Teaching children to be mathematicians versus teaching about mathematics' in *International Journal of Mathematics Education in ~e~ and Technology*, 3, p249-262.

⁵ Klein F. (1939) *Elementary Mathematics from an Advanced Standpoint: Geometry*. Dover

⁶ Mason, I. (1989)

This last point was problematic. I was troubled by an internal conflict that wanted me to be a constructivist and deny the absolutism of "exam syllabi" maths yet could not accept the fallibility of, say, Pythagoras' theorem. A compromise banished the confusion! I (conveniently?) work on a constructivist notion of "coming to know" Pythagoras' theorem within which it is regarded as fallible until the group I am working with establish its viability through explanation and conviction. This has implications for (a) how I would like to teach and (b) an image I have of 'the classroom'. This image is encapsulated in John Mason's phrase "a conjecturing atmosphere" For him, this atmosphere was defined thus:

•.. *when you talk to someone else, you do so in an attitude of acceptance. If you are unsure of what you want to say, then you try and do as much listening as you can, and **if** you are unsure of what you are thinking, then you try and do as much talking as you can. "*

In 1992, Leone Burton⁸ challenged teachers to evaluate their work with a number of questions which "expose the underlying educational philosophy of the classroom and the assumptions of teacher and learners about the nature of mathematical learning. "

Some of the questions were:

- "Is the learning of mathematics a process of induction into a secret society .. in formal, deductive classrooms or a process of ascribing and negotiating meaning .. , making and testing conjectures, as in a collaborative classroom?"
- Does the geography of the classroom reflect delivery of knowledge and skills by a teacher who 'owns' the content, or has the locus of control shifted to groups of pupils ..
- Is the classroom inactive, focused on the production, by the teacher, of items of knowledge .. or does it demand active engagement .. which is personal and fluid, provoking the exchange of interpretation between learners and the engaging of teachers and learners in dialogue .. ?"

Not only did I agree with all of the implied value system in these (rhetorical) questions, I actively modelled my classroom geography and style of working on such a value system.

Good= activity; discussion; groups; negotiation;construction;informality

Bad=focus on teacher; inaction;lack of groups;formality

⁷Mason J. "Mathematics Education: awakening the (re)searcher within" (text of inaugural lecture at the OU on 18/4/91)

⁸Burton, L. (1992) Evaluating an 'entitlement curriculum': Mathematics for all? In *The Curriculum Journal* Vol 3, #2, p161-169

I believed that this was not only consistent with my "working definition" given earlier, but also an essential part of realising it. My current "crisis" is the belief that I was wrong. Leone's description is what I would like to call "fast forward mathematics lessons". I will return to this later! So .. to conclude the "why" section: I accept the process based view of mathematics; I want the "coming to know" the content to be based on notions of fallibility. I contend that this is only possible where discourse is routinely the mode of operation. Discussion becomes a key element in the learning process. If children are constantly trying to make sense of their experience, then I also contend that this is best done communally. I connect the why section with the how by referring to the work of Watts and Bentley~

"In constructivist terms the classroom is often pictured as an arena in which youngsters are asked to consider the ideas and theories they hold for a particular topic, to explore these to some extent, to examine some of their consequences, to listen to and consider the ideas of others and to begin to re-shape their own ideas in order to take account of new factors. "

It seems, then, that we need an atmosphere within which this interplay can take place freely without fear of ridicule and or judgement. Although few teachers would argue for anything other than such an atmosphere, in this context it is essential .. not merely an added bonus if present. But, are classrooms like that? Are teachers?

" .. on the whole, life in school classrooms is conducted in a fairly robust atmosphere, redolent of the normal chiding, teasing and banter that occurs within pupil peer-group and teacher-pupil interactions. On the heavy-handed side, sarcasm, abrasive wit and verbal bullying are not uncommon tools in the armoury of the hard-pressed teacher- or youngster " (Bentley & Watts, 1989)

If life in classrooms is conducted in a fairly robust atmosphere, how do we encourage, allow, and develop conjecturing? From where will come the trust that the offerings of children are done in an atmosphere of acceptance, free of ridicule? Good question!

My starting point was to interview children. I did this in groups with children selected by staff In all around 40 children were interviewed for a total of 5 hours. I had an agenda for the interviews but no structure. I regarded it as a "foray" within which I wanted to explore (a) a very general feel for their experience of mathematics lessons (b) incidents, moments, comments by teachers and peers which had an impact on their willingness to contribute to discussions. The interviews were transcribed and comments were coded to facilitate sorting and classifying.

⁹ Watts, M. and Bentley, D. (1989) "Constructivism in the Classroom: Enabling Conceptual Change by Words and Deeds" in Developments in Learning and Assessment. Murphy, P. and Moon, B. (eds) OU p158-169

Key issues to emerge were:

- Gender: an impression that (a) teachers had different expectations of girls and boys (b) boys and girls behaved differently and (c) boys and girls were treated differently.
- Peer pressure: much of what children said and did was influenced by the reaction, both actual and predicted, of peers.
- Discipline: In most classes it seems that there were actual or potential discipline problems from a minority of children. How staff "controlled" classes was felt to be important in most interviews.
- Teacher behaviour: comparisons were made between different staff and observations were made on perceptions of effectiveness.
- Getting stuck .. and overcoming it.

Some of these issues were discussed with staff There was considerable reluctance to accept children's accounts as an accurate reflection of classroom life. Of particular interest was the mismatch between perceptions of the amount of time spent working. For example, the claim that children found it easy to "not work" for long periods was strongly disputed by some staff

I have also recorded a number of lessons on video. Important differences between staff are emerging. However, descriptions of the lessons would probably be very similar in terms of content, room layout and style ... hence my reference to "fast forward" earlier!

Almost all of the lessons had the same "format":

- Teacher at the front
- Children facing the front
- Activity directed by the teacher: largely through talk (questions / exposition)
- Introduction: pupil work: "whole class + teacher": pupil work (with intervention) : "whole class + teacher"

There were, however, clear differences that the above description would not reveal; some differences that Leone Burton's questions would not "expose".

Work in Progress:

I now have video sequences (generally full lessons) of six teachers. Two of the staff (myself and one other) have been discussing strategies then comparing intended outcomes with "actual" ones. Clearly this does not establish cause and effect. However, I remain interested in the extent to which the classes of the two staff who are explicitly focusing on conjecturing (to varying degrees) ~e operating within a conjecturing atmosphere. John Mason's "Discipline of Noticing" model, with its emphasis

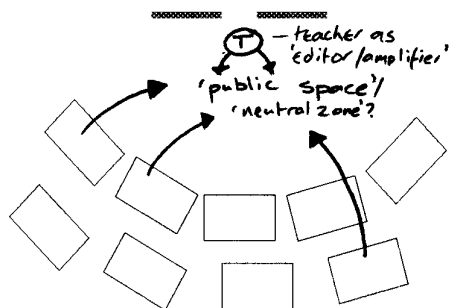
on Reflection and Planning, is providing a framework which offers structure to the at times overwhelming complexity of daily classroom life! Additionally, there is an extra member of staff (maths / learning support) who (a) sees these classes in other subjects and (b) sees other maths lessons by different staff This has provided a useful triangulation point in that this member of staff is not aware the particular emphasis of our planning.

For many staff the issue of classroom management / discipline has been the most sensitive area. Importantly, for most of the children interviewed, it was very high on their agenda. Of the staff I am working with, none have argued that it is unimportant but some have disputed the extent to which children are aware of the dynamics of interaction in terms of discipline.

If I do this, then this might happen ...

"The stricter the rule enforcement, the more freedom I've got .. " was a comment of a motor-cycling ex-colleague describing his management of discussion. "If I know .. and I mean **really** know that all drivers will stay on their side of the road .. especially around comers, then that gives confidence .. it gives me freedom. Strictness and discipline need not be repressive .. it can be the opposite." We thought about children's comments related to discipline and peer pressure. If children are worried about the reaction they get to volunteering answers, then what if we remove the option of

"volunteering"? Would this lead to freer expression?



If I want to create a "geography" of contribution, then what would my classroom look like? I also want to make it clear that I will be "directing" some events. I also want contributions to be worked on rather than the person who made them. A "public space" seemed important. What would be the impact of rearranging the furniture?

A physical "neutral zone,,?10

One way we have made the fallibility of "coming to know" explicit, is to focus on children's accounts of "being sure". By frequently concentrating on articulations of "why" and "I did it this way because", a discussion can be entered into where the subject matter is genuinely negotiated and refined. In this process I have been recording ways in which staff can (or fail to) encourage extended contributions from children. Dave Hewitt developed the metaphor of teacher as editor and teacher as amplifier¹¹. These are phrases I have come to use in explaining different responses by the teacher to

¹⁰ Hewitt, D. (1994) The Principle of Economy in the Learning and Teaching of Mathematics. PhD thesis (OU)

¹¹ Hewitt, D. (1997) "Teacher as amplifier: teacher as editor: a metaphor based on some dynamics in communication." Paper presented to PME21, Finland July 1997.

children when dealing with / engaging with contributions. Although I have a slightly different interpretation to Dave, they remain useful. I have some observations on ways in which a teacher being unintentionally judgmental (a) may inhibit children but also (b) convey the message that a mismatch between the child's current explanation and the teacher's desired one is due to the absolute nature of the content. That is, if the child fails to play the 'think of the words in the teacher's head" game correctly, then the child is wrong.

So .. where now?

Leone Burton's questions? Robin Alexander entered the "whole class teaching debate" with a recent "attack" on David Reynolds et al¹² by asking "what is whole class teaching" and suggesting that the description covers so many different types of activity, it ceases to be useful. Jo Boaler at a recent BSERG meeting referred to 'traditional" v "problem solving" teachers. These are descriptions which I am currently finding less helpful than previously. On fast forward, the teachers I have recorded all look very similar. The reality of each in detail, however, is very different. Unpicking those differences is currently a challenge.

Steve Byatt, Ludlow CE School, May 1997.

¹² Alexander, R. (1996) Other Primary Schools and Ours: Hazards of International Comparison. CREPE occasional paper, Warwick University.