

## WHAT CAN SEMIOTICS OFFER MATHEMATICS EDUCATION?

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*There is developing interest in semiotics as a theoretical perspective in mathematics education. This paper examines semiotics from both a theoretical and practical perspective in order to begin discussion as to how and why semiotics may be a useful perspective for mathematics educators to adopt, or at least consider. A connection is drawn between the work of Vygotsky and Peirce and theoretical development provides the possibility of a consistent philosophical position that transcends Cartesian dualism and offers a new way of seeing. Then semiotics is examined practically from the point of view qualitative methodology. Finally suggestions are made about the role of theory in mathematics education and the possible role of semiotics as one of those theories.*

There is an inextricable connection between signs and mathematics. One might even say that mathematics consist entirely of a complex system of signs, a position that is defensible from within a semiotic perspective. Semiotics is the study of signs and sign functions in all conceivable aspects of message exchange and it concerns the conveyance and development of meaning through all sign vehicles. It is more a point of view than a method and is seen by some (Deely, 1990) in a more general sense, as a framework for the comprehension of the world. Interest in the nature of signs began with Aristotle, writers such as St Augustine and William of Ockham have had something to say on the subject but the term *semiotics* appeared first in 1690 in John Locke's "*essay concerning human understanding*" Semiotics is a point of view of the world that puts the sign at the centre of all human action, it is a doctrine of communication and intrinsic to the semiotic perspective is a dependence on the socio-cultural view of knowledge.

For some time now the mathematics education community has recognised the importance of the socio-cultural context of mathematics learning and such influences have become the object of study (for example see Eccles and Jacobs 1986, Walkerdine 1990 and Evans and Tsatsaroni 1994). Communication is the essential mediator of these socio-cultural influences and language is one (but not the only) sign system which effects this mediation. The role of language, both written and spoken, in mathematics has been investigated by Halliday (1978), Klemme (1981) and Pimm (1987) amongst others, and it is evident from this work that language is instrumental in structuring and developing mathematical thoughts and actions. Nunes (1992) suggests further that specific cultural sign systems structure the organisation of mathematical activities but do not necessarily alter psychological functioning.

Currently there seems to be increasing interest in semiotic issues in mathematics education. A survey of the latest PME proceedings reveals one plenary (Puig 1996) and five research reports (Berenson & Vidakovic 1996, Mcnamara 1996, Radford & Grenier 1996, Redden 1996, Vile & Lerman 1996) connected to this area (this does not take into account the continuing interest in language and other more general socio-cultural issues which could

certainly come under the remit of semiotics), and a number of ICME presentations this year have made reference to semiotics. Why is there such growing interest in semiotics? Is it just another bandwagon or does it (in some form or another) have potential to add to the body of knowledge in mathematics education? In this paper I will consider these and other related questions taking as a context the domains of theory and practice.

### **Theoretical approaches**

The increasing interest in cultural psychology is bringing to the fore the work of Vygotsky (1977) for whom semiotics was intrinsic to his description of the development of higher mental functioning. Vygotsky suggested first that the word and later that tools were the mediators from the intersubjective to the intrasubjective and that concept development began in the social and was then internalised through the action of semiosis. Vygotsky and those working in his perspective put the focus of generalisation of higher mental functioning, specifically the generation of scientific concepts, on the sign. As a unit of psychological functioning he chose word meaning and he gave to the sign the function of mediation, introducing a triadic scheme for meaning making (Vygotsky 1977).

Semiotics did not begin with Vygotsky however, it has two independent roots; one from structuralism and the work of Saussure, the other from the work of Charles Peirce the father of Pragmatism and of existential logic (Ayer 1968). For Saussure signs signify by way of a signifier and a signified, i.e. a diadic action with direct connection. Peirce on the other hand introduced a triadic scheme (Hoopes 1991) with a sign signifying an object only by way of a thirdness<sup>1</sup>. In terms of both theory and analysis Saussurian *semiologists* prefer a synchronic approach, concentrating on the structure of differences in an act of communication rather than the signs themselves. Peircian semioticians propose a more dynamic perception of semiotic acts and analysis is more diachronic, paying proper attention to the history of the genesis of meanings. It would be fair to say that Peircian semiotics does not possess anywhere near the level of sophisticated tools as Saussurian semiology and as a result many of the same tools and much terminology finds its way into both paradigms. Choice between paradigms rests almost wholly on theoretical orientation and Peircian semioticians tend to be more philosophical than practical. Peircian semiotics is more appropriate for a cultural interpretation as it is diachronic in nature, furthermore it is an appropriate starting place for mathematics educators as it has many commonalities with the Vygotskian approach. (For example Maffiolo (1992) draws parallels between the two approaches). For these and a number of other reasons my own work adopts a Peircian perspective.

Peircian semiotics differs from that of Saussure in that Peirce sees the action between signifier and signified or (to use his terminology) sign and object, as triadic. The sign stands for the object in relation to a third element, the interpretant. It is the interpretant which introduces the cultural and historical dimensions to semiotics, but it is not at all easy to understand its role. Vygotsky too considered semiotic action to be triadic yet for him the third element was the mediation means, the carrier of the meaning from external to internal. For Peirce the third element, the interpretant, although it may perform a mediating role is more

than that. Peirce suggests that "[the interpretant] is all that is explicit in the sign itself apart from its content and circumstances of utterance" (Peirce 1906, quoted in Deely (1990) page 26.), and furthermore that the interpretant is a sign.

An example may serve to clarify the situation. Seigel and Cary (1989) describe a situation in which they show to a preschooler a box of toothpaste and ask the child what the writing on the box says. The child replies "brush teeth". In this situation the sign ( the box) stands for the object (toothpaste) in relation to the interpretant (the experience of having frequently brushed ones teeth after having seen the box), this is demonstrated diagrammatically in figure 1. Many other replies were possible, for example the child could have associated toothpaste with going to the dentist or going to bed, the sign could have indicated a number of different meanings to the child by providing for further context dependent interpretants.

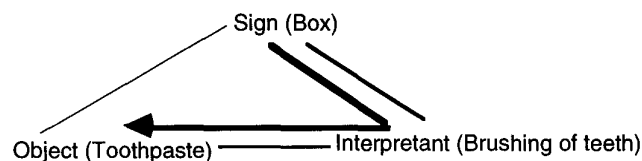


Fig 1

Interpretation of the sign may only take place by way of the interpretant, any direct connection between the sign and the object that is not recognised will remain devoid of 'sense'. The interpretant carries with it the socio-historical positioning of the interpreter and performs the functions of mediation and meaning-making, it is the single most important aspect of semiosis, for without it semiotic acts would have no meaning. The Interpretant of a sign may become a sign in a new act of semiosis for which a further interpretant is needed in order to make sense of that act. In this way a process of "unlimited semiosis" (Eco 1979 p.69) begins evolving and developing the meanings of the sign diachronically, as a chain of signifiers<sup>2</sup>. The very function of semiosis opens up a number of different meanings by providing for context dependent interpretants. Sign interpretation is bound up with everything we do, will do or have done and any given act of semiosis will be inserted into this network of experiences, this network accounts for the variation of individual interpretants. A sign is therefore not simply a substitute for a thing but contains some aspect of the meaning of that thing to us.

The introduction of the interpretant into the meaning-making equation also brings into question the nature of experience. Deely (1990) suggests that the semiotic universe divides the content of experience into three areas. The physical, the objective and the subjective. The objective is that which is experienced, the physical is that which is existent regardless of its being experienced and the subjective exists only in relation to the action of semiosis in which something that is objective is recognised and becomes a sign. The physical environment is thus experienced by the subject through a network of objective relationships which are then reconstituted by a subject in a subjective domain into an objectively shareable world.

Through the interpretant the action of semiosis defines a three dimensional meaning-space which transcends subject -object dualism. Semiosis takes place primarily in a social space and interpretants may be thought of as *cultural units* (Eco 1979) which both enable and are a product of a shared cultural understanding.

Vygotsky (1978) saw the development of higher mental functioning resulting from progressive control over semiotic systems of increasing demand, in this description thought is "nothing more than the self-appropriation of the cultural space" (Maffiolo 1993). Vygotsky considered the word to be the cultural unit and saw the role of the word as central to the cultural line development and specifically to concept development. The work of Becker and Varelas (1989) has investigated the Vygotskian notion of semiotic ally mediated development. They conducted a study of counting and place value activities with increasing degrees of semiotic demand noting that as development occurs the sign-object connection gives way to a sign-sign connection, with signs becoming more opaque as development progresses.

In his description of the development of scientific concepts Vygotsky (1977) invokes a semiotic process by which "the relationship to an object is mediated from the start by some other concept. The very nature of a scientific concept implies its position in relation to other concepts" (Vygotsky 1977, p.93). Scientific concepts are concepts that may be learned in a school context, things such as number and family relationships, and their nature is such that they depend upon other concepts (which from a Peircian point of view would be thought of as signs) for their meaning. This is consistent with the Peircian notion of unlimited semiosis (Eco 1979) in which interpretants become signs which in turn become elements of further semiotic acts.

The works of Peirce and Vygotsky independently propose semiotics which transcends the subjective-objective divide through notions of thirdness. The Interpretant provides the three dimensional meaning-space where the intersubjective and intrasubjective meet to make meaning. Rather than consider the interpretant as the mediator from external to internal (as is the implication in the writing of Vygotsky 1977, Eco 1979 and Ohtani 1994) the role of the interpretant is extended to be a space in which meanings are constituted and reconstituted both publicly and privately.

With Peirce and Vygotsky, [we] recognise the subject as a sign and the sign as a cultural object, we place ourselves in a dialogical perspective, where thought is elaborated by and in the activity of communication, where cognition is constructed by and in the share of speech codes and contexts. This is in fact the opposite of traditional epistemology, which assumes that human beings use signs to communicate the thoughts they previously and individually constructed according to natural and/or universal processes. (Maffiolo 1991 p. 491).

My own work has been concerned with the evolution of a theoretical framework which synthesises diachronic Peircian semiotics with the developmental aspects of the work of Vygotsky into a coherent theoretical perspective with sound philosophical underpinnings. Developmental semiotics (described in more detail elsewhere: Vile 1996, Vile and Lerman 1996) has evolved to incorporate re-defined and refined notions from the work of Peirce and Vygotsky. Central to this perspective are the assumptions that mathematical meaning-making

is social in nature and that any account of mathematical meaning-making should take account of development.

There are two main elements of the process of developmental semiotics. The first, from Peirce and Deely, describes the way that a cognising subject is in possession of a network of experiences into which any semiotic act will be inserted and an interpretant (and hence a meaning) will be made. This network of experiences is built up over time with each new experience affecting the whole network. The second, from Vygotsky (1977), puts forward the idea of a developmental process in which the external social factors affect the level of mental functioning and force the transition from elementary to higher mental functions. Meanings are made through the sign-object-interpretant action, the interpretant being the element of thirdness that transports a physical, sign-object brute act of secondness into the objective, social and, subsequently, intrapersonal plane. The interpretant is the indicator of the meaning. Each interpretant itself becomes a sign and may enter, at the level of secondness, into a further act of semiosis. In this way an unlimited semiosis tending towards sign-sign (symbolic) functioning will be built into the network of experiences. The overall result will be a developmental shift towards the more abstract with the object referent moving more to the background and sign-sign functioning becoming foregrounded (Becker and Varelas 1989)

As well as providing a theoretical perspective, a lens through which to view mathematical meaning-making, developmental semiotics provides a vocabulary for theoretical and practical description and discussion of actual and virtual meaning making processes. Important in practice are notions of *semiotic demand* (relating to the level of opacity of a given sign) *sign-sign foregrounding* (relating to the degree of generalisation of a sign held by an individual and measured by their ability to access a sign with a given semiotic demand), and (sign) *meaning* (or interpretant). I would like to suggest that developmental semiotics may be fruitful both in description of meaning-making and as a tool for qualitative description of the meaning-making process.

### **Semiotics as a qualitative methodology.**

Semiotics as a philosophical perspective is concerned with the world as a web of signs (Deely 1990) and semiotic analysis is concerned with the reading of those signs. In this respect research within the qualitative paradigm (symbolic interactionism, phenomenology, ethnography etc. ) with its search for meaning in action and culture could be considered to fall within the semiotic perspective. Shank (1995) suggests that:

Semiotic theory can help expand the conceptual and practical domain of qualitative research by serving as a philosophical foundation of the discipline, thereby allowing qualitative researchers to build upon a set of ideas that powerfully extends the aims and goals of their research. Qualitative research in education can help expand semiotics by serving as a source of empirical research and findings, thereby helping move semiotics away from its current near total preoccupation with theory and into a state where empirically determined issues play a more important and visible role. (para. 2)

Semiotics does possess a number of tools for analysis of qualitative data but as Shank says the emphasis has been more on theory than practice.

Semiotic chains, semiotic clustering and semiotic squares are amongst the most

- 1) **What are the important signifiers and what do they signify?**
  - What is the system within which these signs make sense?
  - What connotations seem to be involved?
  
- 2) **What is the syntagmatic structure of the text<sup>3</sup>?**
  - How does one unit (action, part of the text) relate to another?
  - Are there formulaic features that have shaped the text?
  
- 3) **Paradigmatic analysis**
  - To which class of paradigms (medium; genre; theme) does the whole text belong?
  - What might the text have been like if it had formed part of a different genre?
  - What paradigms are noticeably absent?
  - What paired opposites seem to be involved (e.g. nature/culture)?
  - Is there a central opposition in the text?
  - What psychological, social and political import do these oppositions have?
  
- 4) **What semiotic codes are used?**
  - Which conventions are most obvious in the text?
  - Which codes are specific to the genre of text (writing, acting, speaking etc.)?
  - Which codes are shared with other genre?
  - What cultural assumptions are called upon?
  - What seems to be the preferred reading?
  - How far does this reflect or depart from dominant cultural values?
  - What alternative readings seem possible?
  
- 5) **Intertextuality**
  - Does it allude to other genres?
  - Does it allude to or compare with other texts within the genre?
  - How does it compare with treatments of similar themes within other genres?
  - What other contributions have semioticians made that can be applied to the text?
  - What does a purely structural analysis of the text downplay or ignore?

(Box 1)

common techniques used in semiotic analysis. Manning (1987) and Feldman (1995) outline these and other tools arising from a structuralist semiotic perspective. An excellent article for beginning semioticians by Chandler (1996) identifies a number of aspects of semiotic analysis drawn from both diachronic and synchronic points of view. In a section entitled "D.I.Y" Semiotic analysis he suggests a number of guidelines for the analysis of media and text which I have adapted somewhat here for the sake of generality (Box 1).

As an example of the use of these criteria consider an analysis of the following "text" which was constructed by a student, who shall be known as Becky. The data was collected during a case study in the area of algebra.

①  $3y + z = y + 6$  you don't want the z there, so you take it away from 6

$3y = y + 4$

$2y = 4$  you don't want the y there so you take it away from 2y.

$y = \frac{4}{2} = 2$

you are trying to find 'y'.

②  $5(1-Q) = 3(5-Q) - 2$  you do  $5 \times 1 = 5$   
 $5 - 5Q = 15 - 3Q - 2$   $5 \times Q = 5Q$   
 $3 \times 5 = 15$   
 $3 \times Q = 3Q$

$5 - 5Q = 15 - 3Q - 2$  you don't want this.  $-3Q$

$2Q - 5 = 13$   $13$  don't want this.  $+5$

$2Q = 18$

$Q = \frac{18}{2} = 9$

a) What are the important signifiers and what do they signify?

This text is replete with signifiers, some from the system of mathematics and some from the system of the English language. There are two different sign types, symbols (X, 2, don't, etc.) and indexes (the arrow) and it is interesting to note that the index is used in this case only when reflecting or describing, not when doing.

b) What is the syntagmatic structure of the text?

Algebra by its very nature has an intrinsic "grammar" and structure. Becky's work is structured along these lines, work is sequential and reflections describe the processes passed through from step to step. In a sense Becky is telling the story of how she solved the equation and in a syntagmatic sense this is the same story for both equations: Equation is written; Equation is expanded to a recognisable form; equation is manipulated to the form  $ax=b$  by way of addition and subtraction of appropriate terms; unknown is stated. This is perhaps a basic protocol or genre for the solution process that Becky is using.

c) Paradigmatic analysis

This text perhaps belongs to a syntactic solution paradigm, but not entirely as there is a sense in which her manipulations could be endowed with more meaning, presenting a syntactic/ semantic complementarity. Paradigmatic analysis usually proceeds by identifying oppositions in the text, I have identified a complementarity yet an opposition does exist in this text between the knower and the known (or in this case the unknown). Becky knows that she is 'trying to find y', but she does not know what it is, although y exists as a known by virtue of the structure of the equation. For Becky it seems that in this instance the value of y is to be uncovered, in a sense it is known and in a sense it is not.

d) What semiotic codes are used

There are codes specific both to mathematics and to the medium, in a sense this sort of mathematics would be very difficult to do without visual coding, but by the same token a computer screen could generate a different use or meaning of the codes used. Clearly Becky is using signs that are culturally accepted as appropriate, but it is not clear whether she shares the same code as others. All we have to go on is the fact that the protocol is followed accurately. One reading of the text is that Becky is acting meaningfully in a context in which she feels comfortable. Another is that she is senselessly rule following.

*e) Intertextuality*

This analysis can be only really be achieved through comparison with other extracts from other students solving the same equation. In addition there are elements of the text which have been ignored relating to the diachronic nature of Becky's learning. A full intertextual analysis is only possible if history is included in the equation.

Overall this analysis is structural and synchronic, yet it has brought to light a number of different elements of the text which perhaps were not explicit. However this type of structuralist analysis has been criticised (Deely 1990) because of its synchronic approach and lack of consideration of cultural and societal issues. It is still valuable (but perhaps not as revealing) to construct the analysis of a text in isolation. An alternative analysis must consider the text as one part of a history of connected texts and experiences. A consistent set of tools for this type of analysis does not yet exist but one may begin to interpret the text from within a developmental semiotic point of view.

How might one begin to analyse the extract shown above from the point of view of a developmental semiotic? One of the basic tenets of developmental semiotics is that extracts like this should not be taken in isolation and so to give a brief history of this student's algebraic action would be appropriate and perhaps revealing. Becky had a very successful beginning to her experiences with linear equations. Equations such as  $x+5 = 7$  were solved with recourse to trial and error methods like the ones shown in the following extract (which does not come from the pen of Becky, but illustrates the general principle and will be used as a comparative example later on in this discussion).

Solve for A  $7 + 2A = 4A - 11$

Handwritten work showing trial and error for solving the equation  $7 + 2A = 4A - 11$ . The student lists values from 1 to 15, crossing out most of them. The value 9 is written as the answer.

Equations such as the one above were not successfully solved by Becky as she was forced to progress from trial and error methods by the increased semiotic demand of the equations into an attempt to manipulate symbols to find the value of the unknown. Equations were increased in semiotic demand until we reach the stage about which Becky is writing in the extract above. By this stage she has become more successful. She explains how to solve an equation with the unknown on both sides. This is an equation with a high degree of



semiotic demand. In other words it is an equation in which the signs do not stand directly or transparently for existent objects (rather they represent a chain of signification from protonumbers to arithmetic) and as a result potentially may be thought of as objects in their own right. Becky states "you don't want the Y there" ( I was not able to ask why not although I would have liked to) and demonstrates her ability to "take Y away from 2Y", in order to isolate the unknown, that is to treat the sign as an object in its own right (Pimm 1995). The referent of the sign (whatever that is to her) has been backgrounded and the connections with other signs has been foregrounded giving rise to a degree of sign-sign foregrounding that enables Becky to manipulate, solve and reflect upon the equation.

The second equation is more problematic. There is a higher degree of semiotic demand, this equation consists of a complex string of symbols with associated rules of manipulation. The problem occurs in the third line where Becky misses out a number of steps and writes down an identity which does not follow from the previous line. She produces the answer 9 but does not substitute to check that this satisfies the equation. In treating the sign as an object, Becky has become over-confident and as a result has made an error in "mental algebra" . The real problem is that she seems not to see the connection between the symbol for the unknown and the number which is the answer to the equation. In a pre-study interview when asked what the x stands for in the equation  $x+2=5$  she said, "x is something you don't know what it is".

Compare this with the example above in which A is established as 9 by trial and error method. This student does not possess the required degree of sign-sign foregrounding to consider the unknown as an object and to manipulate it, but she does possess a well developed meaning of the nature of the unknown - as a number. The equation has a high degree of semiotic demand yet this student is able to solve it by virtue of a more transparent (less developed?) notion of the sign.

This type of analysis can be applied to individual cases and to larger case studies. It is not at all difficult to see how such analysis, with its search for meaning and focus on the sign, has the potential to inform future practice. In the case of Becky steps must be taken to introduce the notion that X stands for a general number, and in the case of the second student steps must be taken to force the construction of X as an object in its own right, perhaps through the introduction of equations with non-integral solutions.

It is not the answering but rather the asking of questions from within a semiotic perspective which may be revealing, for the focus of these questions is the reading of a text and of signs within that text. Educational researchers using semiotic analysis need to become good readers of signs, but that is not to say that the readings or interpretations that they make describe with any degree of "trueness" the underlying structures, meanings or connotations manifested in those signs. Questions may be raised about the value of semiotic analysis if one accepts the relativity and subjectivity of the techniques. But this is not problematic if one sees semiotics not as a methodology for extracting descriptions of the way things are but rather as a way of "illuminating" (Feldman 1995 p.39) relationships and structures, that is as a toolkit to enable the creation and evolution of interpretations that make new sense of data in a way

that develops our own understandings of phenomena and requires us to ask and attempt to answer new questions. In providing a set of empirical tools for the analysis of data semiotics presupposes theoretical tools of equal importance which carry with them a register, a vocabulary, a way of seeing and talking that will enable discourse and the creation of interpretations.

Qualitative research from a semiotic point of view is not aimed at extracting facts but in describing the world through the action and interpretation of signs. I like to use the metaphor of semiotics as a lens through which to view and interpret the world and a scheme for research based upon this world view may provide a systematic, empirical methodology with a strong theoretical underpinning that may prove fruitful and revealing in all areas of social research. I have shown one example of how this type of analysis may be applied in research but one may ask whether there is indeed any use of semiotics in the classroom.

Teachers have so much to cope with nowadays, so many new curriculum documents and ideas, that they hardly have time to think about anything else apart from teaching and administration. It has been suggested (Scott-Hodgetts 1990) that teachers should be interested in research in mathematics education and that indeed they should actively be engaging in research in their own classrooms. Lerman (1990) introduces the notion of the reflective practitioner to the classroom and in this sense it is clear that every teacher engages in research of some sort, perhaps in testing new materials or teaching ideas or perhaps in interpreting pupils actions. Scott-Hodgetts (1990) suggests that, for the teacher, involvement in research helps develop a more acute awareness of the activities around teaching and learning in the classroom. It is in this I that I believe semiotics has a part to play.

As an interpretative framework with focus on meaning rather than knowledge and with a body of pre-existing theoretical and practical ideas semiotics has potential to provide insights into the ways in which students make meaning in the mathematics classroom. What semiotics does is change focus of interpretation and inquiry to the sign. In the most simple case teachers can ask of themselves: what does that sign mean to that student in that setting? Questioning, observation and experience may give rise to answers which would lead to solutions, to ways of helping a student redefine for themselves the nature of the signs that they are using in a way that will help them cope with signs of higher semiotic demand. Additionally a semiotic perspective raises questions about the nature of teaching and learning. With focus on the sign teaching is viewed as a process of facilitating meaningmaking rather than imparting knowledge and this in turn brings to the fore notions about the relative nature of our subject. Semiotics may offer teachers both a valuable interpretative framework and a challenge.

Of course if one subscribes to this view one must accept that mathematics consists entirely of signs, that is it is a semiotic system (Rotman 1995) and that, for what ever reason, it progresses by producing ever more abstract signs with ever more increasing semiotic demand and that a teacher's role is in part to give access to those signs and that semiotic system. If these premises are accepted then it becomes imperative for a teacher to begin some sort of semiotic analysis on the mathematics to which she intends to expose the students and

of their individual reactions and actions and hence meanings. There is no need (for example) to ask questions about the syntagmatic and paradigmatic aspects of the text, but such terminology may prove useful when experiences and interpretations are shared with others. Semiotic analysis occurs in the classroom today, it is just not labelled as such, and a more systematic approach, such as that presented in this paper may be more helpful. The aim always is to read the signs and then, if appropriate and efficient, effect change.

Semiotic analysis is not confined to the mathematics aspect of the mathematics classroom. One of the key elements in semiotics is the experiences that are brought to each semiotic act (Deely 1990). Students bring experiences from home, from the playground, from the TV etc. into the classroom And these experiences will have an effect on the way in which they interpret and act with the signs all around them (even the mathematical ones as Walkerdine (1989) has shown). Semiotic analysis of small classroom incidents could give clues to a larger socio-cultural picture.

### **So What Can Semiotics Offer Mathematics Education?**

Before this question can be fully answered the role of theory in mathematics education must be considered. I suggest that theory is not only useful but essential in our attempts to make sense of the world in which we exist, and of the observations that we make. Feyerabend (1975) contends convincingly that observational statements presuppose theory, that although natural observations may be made, attempts to make sense of them, describe them and share them require some kind of theory. Clearly there are strong grounds for the acceptance of this view in both science and social science and the implication is that in order to observe and interpret classroom phenomena some theory is necessary. Additionally the sharing of a common theoretical perspective would perhaps provide a context and vocabulary for discussion and interpretation within a community. Mathematics education has within its bounds a number of such theories and mathematics educators have the possibility and facility to pick and choose, and use these theories in acts of observation, interpretation and prediction. The development of such theories aims to improve their ability to provide fruitful observation statements and to make new and useful interpretations and is an important aspect of mathematics education research in itself contributing to the evolving dialectic of theory and practice. Semiotics is such a theory, in mathematics education terms it is only in its embryonic stages but already it is (as I hope that I have shown here and elsewhere) able to contribute both theoretically and practically to the general body on mathematics education research and practice.

I would like to suggest that semiotics may be of use for mathematics educators (and also for other researchers in qualitative paradigms) in both a reconceptualisation of theoretical perspectives and in terms of empirical methodologies. Theoretically, semiotics may offer a description of the meaning-making process that accounts for the socio-cultural nature of experience and, avoiding Cartesian dualism and with it problems with intersubjectivity (Lerman 1996), provides an alternative description of the social/personal interface. Empirically, focus on the role of the sign, and in particular the nature of the interpretant, may

assist in the understanding of the meaning-making process in specific contexts. Continuing work would necessarily build on work carried out in specific semiotic systems such as that of Klemme (1981), Pimm (1987), Rotman (1988), Stage (1991) and Nunes (1992), and may serve to unite such work under a comprehensive semiotic framework. I hope that the discussion of the theoretical aspects of a semiotic point of view will be sufficient to generate interest from the mathematics education community in the possible application of a semiotic framework to the description of meaning-making in mathematics.

In the current climate, mathematics educators are moving towards a description of mathematical meaning-making that involves society, culture communication and context, and many are moving further to investigate the role of the sign in mathematical meaning-making. Semiotics, at least in versions presented by Peirce and Vygotsky, exists as a well developed point of view of the world that embraces all of these elements in a way that transcends traditional philosophical objections and as such may prove (and indeed is already proving) to be a fruitful perspective for the description of meaning-making in, in particular, mathematics education.

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1 This relates in some senses to Vygotsky's notions of mediation

2The term "chain of signifiers" has been used by Walkerdine (1988) in a very different sense to that in which it is used here. For Walkerdine this chain of signifiers represents a mainly synchronic shift in domains, here I use it in the sense of an historically developing system of dependant and compressed meanings from sign to interpretant to sign etc ..

3Text here relates not only to written text but any system of semiotic action, a text could be a classroom dialogue, a piece of writing or a physical action.