ASPECTS OF PRACTISING IN SCHOOL MATHEMATICS

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This preliminary paper briefly outlines an analysis of a range of purposes of practising mathematics wIthin a school context, defining three main objectives of practising. A more detailed analysis and discussion of the research into these objectives and the author's categorisation of practising will form the basis of a paper to be submitted to BSRLM later in 1997. *The current paper reviews the historical context, as waves of educational thinking and practice have spread nationally over the past forty years. Within this review, such key aspects as 'practical activity', 'pace " 'investigation' and 'schemes' are briefly discu.'lsed. In the second part, some illustrative indications are given of the problems of engendering change, and of the kind of assumption held by the author. The notions of 'productive practising' as developed in Germany and of 'green practising' as developed by the author are introduced and illustrated, then related briefly to the author's categorisation of practising.*

'Practising' is a broad term encompassing three active processes through which

- partially acquired concepts are strengthened (a.k.a. 'consolidation')
- acquired knowledge, both factual and that relating to methods and approaches, is transferred from short-term memory to long-term memory (a.k.a 'memorising')
- embryonic skills are sharpened until problems to which they are applicable do not normally require a problem-solving response, but instead become routine (a.k.a. 'routinising't

In England, the enacted, and in some cases also the espoused, beliefs of the great majority of teachers of school mathematics and arithmetic up to about a quarter of a century ago appear to have been that knowledge of number and of methods of operating on and with numbers become fixed in our minds through repetitive practice, and through our submitting to speed recall and speed calculation tests tests with an inbuilt strong competitive element. This belief not so much carne into question in the early 1970s as fell into some disrepute. The attempts made to humanise mathematics educationS, along with other wider changes of attitude within education - sometimes labelled as 'progressive' - tended to make such beliefs and practices much less likely to be openly espoused by practitioners. In part this change became bound up with evidence then surfacing about negative attitudes to the subject within the adult population, which Sewell[1] and Buxton:! illuminated in complementary ways. The connection was made between such attitudes and the classroom experiences of 'boring' practices and drills, and of repeated failure at speed-based mental tests within the personal histories of many adults. The persistence of such attitudes into and throughout adult life for a significant proportion of the population at all levels of actual competence and intelligence was such that a cycle had become established, making it problematic to foster more positive attitudes in succeeding generations without etfecting some significant break with the past.

While the tirst wave of reform of the modem era brought teaching aids with which teachers could demonstrate mathematics practically, it also brought a reformed curriculum involving at least some of the mathematics that had originally been developed within the preceding 300 years. The origins of the second wave can be found in the work of Jean Piaget, whose interpreters brought into classrooms especially in primary schools - a range of practical activities within which learners manipulated materials directly. The leading primary mathematics 'scheme' of the late seventies and early eighties was subtly subtitled *'a development through activity'* so encompassing the promise both of a sound developmental basis through applying Piaget's research findings and of a practical activity-based approach. A third wave was sent on its way, perhaps inadvertently, by the advent and later report of the Cockcroft Enquiry.3 In its key paragraph 243, it was the term 'investigation' which provoked the most obvious active reforming zeal. Of course the idea of 'investigation' was also actively reinterpreted - largely in ways that allowed for its inclusion with least disturbance to the rest of the existing mathematics curriculum. Now by labelling it as 'investigations', a broader approach was not developed and any truly investigative activity as remained became cabined within either a teacher-led approach or within a specific time-slot where it could be kept separate from the rest of mathematics almost as if a Cockcroft 243 tithe had to be paid!

In the mainstream curriculum beneath the surface many features had not changed significantly. Despite curriculum reform and the practical activity approach, abstract number was still practised and tested so much that other modes and areas of mathematics were consequently squeezed. The author carried out an analysis in 1987 by a case study in a junior school who were piloting a new published 'scheme'. It found that while a page-by-page content analysis of the 'scheme' materials revealed around a 40% emphasis on non-contextualised number operations, in practice this aspect occupied more than 70% of pupil time in lessons. Parallel analyses of other 'schemes' in practice have since confirmed these kind of proportions. The continued sales of such 'schemes' strongly indicates a continuing preponderance of non-contextualised . number work' in the primary curriculum.

In the junior school case study cited above the majority of pupils worked 'at their own pace' through the published pages - a pace that appeared to depend more on their motivation than on their potential, so that a danger of failing the undermotivated was apparent. There was no clear distinction made by teachers or by the scheme between fresh learning on the one hand, and consolidation and practice on the other, so that episodes of teaching had become incidental and to an extent accidental in that they were often sparked by a reco&YJIition of common errors occurring as a series of pupils presented with similar problems, or by a sufficient level of teacher frustration with an individual or small group. Either way it was commonplace for all pupils in the class to have to attend to the fresh burst of teaching, or the near repetition of prior teaching, regardless of their relative competence or level of understanding. An apologia for the 'own pace' approach at the turn of the 1980s was typified by an analogy - *"it is not possible to drink from a high-pressure hose"* - which requires some attention. The likening of 'drinking' - via 'taking in' - to 'learning' may have some credibility, though it is not the richest available metaphor for the learning process! However, in tenns of its application to practising - as defined above - it is difficult to see the analogy holding at all. What it obscured, importantly, was the positive place of appropriate pace in two aspects of practising - memorising and routinising.

The dominant fonn of practising as enacted through the 'scheme' was typifiable or parodied as:

Zigs 32

This is how to do zigs:

[within this space would be a demonstration example]

Exercise One

1	1 zig 5	2	2 zig 0
3	2 zig 3	4	5 zig 2
5	4 zig 7	6	6 zig 6
7	10 zig 2	8	10 zig 4
9	3 zig 30	10	4 zig 21
11	2 zig 11	12	9 zig 8
13	15 zig 12	14	18 zig 13
15	18 zig 31	16	67 zig 7
17	52 zig 5	18	81 zig 15
19	77 zig 19	20	100 zig 10
21	44 zig 161	22	314 zig 11
23	222 zig 53	24	1000 zig 2
25	321 zig 322	*now **investigate** zig some more!*	

The level of 'recognition' evoked in large gatherings of teachers of mathematics by this illustration lends weight to the view that, even in recent times, too often practice of skills is left to the pedagogical equivalent of the horseless carriage, with the teacher walking in front holding a red nag;' answer book.

As Seiter" counsels "Primary school practices cannot be changed on a large scale, if no alternative perspectives on the practising of skills are offered to teachers." There did not appear to be much focus on this in the late 1980s in England, when this research began as a response to the case study previously cited. There are a number of rather more motivating means for practising mathematics effectively, so if Seiter is correct, these have not been' offered to teachers' in ways that they have felt able to adopt. BrowneU[5] in 1956 said that no essential progress had been made in appropriately balancing the focus on meaning and the focus on skill. The position forty years on remains unlikely to match up to his specifications.

Assumptions lie behind decisions as to how practising should be effected. The author assumes that practising related to 'memorising' should involve pupils in working as fast as their capability allows, as that is a factor in causing learning to move into long-tenn memory. The motivation for working fast should come from the design of the practice activity. Gillian Hatch[6] has pointed out, for example, that *"some games generate an 'unreasonable amount' of practice,* " a phrase which indicates the near optimal pace sometimes achieved. Other example activities have been indicated by the author in several articles[c] during the last five years.

SeIter" compares two different ways of practising, and discusses the notion of 'productive practising.' He describes the previous predominance of 'pages of bare sums' in Gennan primary schools and the movement there to make the practising of skills more motivating, supported by the analysis of Wittman'. SeIter compares the 'same' page of sums, if correctly solved as leading to the connecting of crosses to produce' a nice sailboat' picture - just as Wittman, and later SeIter' also, have described the production of 'coloured dogs' from pages of sums. He offers an approach in which the

practising of skills is a 'crucial part of the learning process' and where there are both 'material based' tasks and 'coherent' tasks. Wittmann & Muller (1992)' reported on examples of 'productive practising' including:

12	123	234	345	456	567	678
+432	+432	+432	+432	+432	+432	+432

333	444	555	666	777	888	999
-321	-321	-321	-321	-321	-321	-321

A pupil, having solved these, wrote: 'It is always III more. In both series the answers are 111 more. The number to add is always the same and the numbers you have to minus as well. '

An illustration of a form of practising developed strongly within this research, and referred to as 'Green' by the author[10], is given below. It uses the terms **sum, difference** and **product,** though compromises of language forms are made where this would deflect from the activity being taken up fully by teachers. Pupils - here in Year 4 - often working as paired partners, were introduced to the idea that given two numbers, it is possible to find their sum, difference, and product. They are given some simple examples, then asked to choose their own pairs of numbers. At a suitable point pupils were led to recognise that the calculations they had made were three clues to their two numbers. They began to swap their output, but retained their original numbers~ therefore they were faced with problems to solve. A rule was introduced that problems must carry a code ofM, **P,** or C to show how they had been constructed: mentally, on paper, or with a calculator. The scene was set for pupils to work in a 'green' way, generating their practice from within their own resources. As in any green scheme there was some stage management involved, which initially fell to the teacher. D

Martin and Vicky began creating their problems gently, by choosing first 2 and 1, then 7 and 8 as their numbers~ this was what was expected of them. However they amazed their teacher with their third example, by choosing 12 and 17, and continuing to calculate the sum, difference, and product in their heads; this was well beyond any mental calculation that they would have been set. Vicky explained that they had worked out the product by talking it over, during which they realised that ten 17s would be a 170, so they worked out two 17s as 34, then added 30 to 170, then the other 4, to make 204. Ironically, they made the difference 4 (not 5), but this came out into the open when other pupils were solving their problem, and reported difficulties to them.

Meanwhile, Anna, working alone, chose 40 and 30, quickly producing a sum of 70 and a difference of 10. She resorted to a calculator to find out that the product was 1200. Later she presented. her problem to be solved mentally. When 1 challenged her on this, she said it was 'OK' since she reckoned she could 'now do it in her head', and she solved two similar probl~ms correctly for me to demonstrate this: 30 x50 = 1500 and 20 x40 = 800. These Year4 pupils were all of moderate attainment, and this was their first 'green' session. A relatively high-achieving pupil, Gareth, made a natural transition from mentally tackling pairs of numbers and their 'yield' eg(6, 19 -> 25, 13, 114), (25,19 -> 44, 6, 475) to using a calculator for eg (55, 19 -> 74, 36, 1045) but refined his record of this by indicating that only the product involved use of calculator! He made his problems with great confidence; he gave one of his original numbers *(a/ways the partner ofhis persistent 19s)* and the sum and difference, so that solvers needed to find his second number and the product. In this he judged his classmates abilities well.

There are several different **types** of practising possible within classrooms, yet of this mnge just a few predominate. The first three commonnlace tvnes identified within this research are:

plain	pupils are presented with non-contextualised, author/teacher generated examples.
'purl'	pupils are presented with pseudo-contextualised author/teacher generated examples, often on 'decorated' pages.
self-checking	although examples are often as above, pupils are presented with a structure within which they can determine on their own what is their level of success; these include Selter's sailboat and Wittman's coloured dogs, but also other activities that do not have such apparently negative features. For example, 'Loop Cards' games[E], invented by the author, an exemplar of which was presented at ICME8, are games founded upon self-checking.

The two developmental categories of practising that are far less commonplace are given below, and it is these which will receive the most attention and illustration in the forthcoming paper:

patterned	pupils are presented with examples within which there is an underlying mathematical structure - a pattern in whose recognition the structure may be perceived; this is *essentially* what Selter calls 'productive practising'.
'green'	pupils are presented with a structure in which once they have achieved some degree of familiarity, they generate problems for each other; graduation of difficulty and differentiation become their concerns

NOTES

A significantly, 'routinising' also means 'to lose interest in' and this can be seen in action when skills are deployed to a purpose within which the focus and interest is instead invested.

B for example, see David Wheeler's *'Humanising Mathematics Education'* in Mathematics Teaching

C for example, *'Mental Calculation'* in Strategies 1,1 (1990), *'Broken Numbers'* in Strategies 4,3 (1993), *'All in the Game'* in Strategies 5,5 (1994)

D for a more detailed discussion of such practising, see my forthcoming paper.

E 'Loop Cards' (by the author) is a book of mental calculation practising games, written in 1986.

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