INTERACTIONS BETWEEN VISUALISATION AND SYMBOLIC SENSES

Ian Malabar and Dave Pountney

School of Computing and Mathematical Sciences
Liverpool John Moores University

Abstract

Research at Liverpool John Moores University into effective use of computersff.T. in mathematics teaching has included the influence that enhanced computer visualisation techniques has on pupils' learning, and in particular how this visualisation interacts with symbolic representation and manipulation skills to the benefit of the latter, if at all. A research experiment which attempts to quantify the influence of visualisation in the learning about graphs and functions is outlined. This involves the design of an interactive package which links graphical and symbolic representations in such a way that it reinforces students' conceptual understanding of functions, and at the same time enhances their ability to visualise.

1. Introduction

Mathematics education is currently at an evolutionary stage with regard to computer technology which can now provide alternative approaches to the teaching and learning of mathematics at all levels. Mathematical software such as DERIVE, CABRI-GEOMETRI and MATHWISE, and advanced graphics calculators such as the TI92, coupled with advances in multimedia technology, are beginning to impact significantly on mathematics teaching, both nationally and in Europe, and yet attempts to measure the benefits to the learning process are relatively few. This research endeavours to explore the impact of such technology on the cognitive aspects of mental images created by learners and the motivation effects of such approaches to learning for both 'more able' and 'less able' students. This research is building on current investigations within the School of Computing and Mathematical Sciences at Liverpool John Moores University (see for example Pountney [6], Townend [8]).

2. Aims and Objectives of the Research

The overall aim of this research is *"to measure and evaluate student learning of mathematical concepts via computer generated dynamic visualisations"*. The dynamic educational package concerned with the graphical representations of functions, described in Section 4. of this report, is aimed at students studying A-level mathematics and first year mathematics topics at degree level. This example found in upper secondary and Higher Education syllabuses has been chosen as it is this type of subject matter on which our dynamic approach should have a more considerable impact. It is anticipated, however, that the conclusions reached relating to this approach to teaching and learning will prove beneficial and applicable to all age-groups and levels of mathematical ability.

Mason and Heal [4] state that 'the pictorial enables aesthetic appreciation and invokes holistic visual processing directly, whereas written symbols must be processed sequentially, and require construction of a rich inner imagery to achieve the same effect'.

Through the design of experiments, we will establish via case studies the truth or falsity of the hypothesis that the use of dynamic visualisation improves the understanding of mathematical *concepts* (as opposed to processes).

3. Rationale for the Research

One of the key reasons for undertaking such research is that there is a real problem that needs addressing (see for example [1], [5], [7]). Many students are failing, or even avoiding taking, mathematics. Positive steps need to be made to rectify this disturbing situation.

School Ie avers are becoming increasingly ill-prepared for further study [7]. Virtually all science subjects include some mathematics, and it is apparent that students do not have the necessary preparation for such a mathematics content. A trend away from combining mathematics and science A-levels as a coherent course of study has developed [7]. Students are now often encouraged to choose a 'more balanced' course of study, containing subjects from both arts and sciences.

However, trying to seek a broader education can create problems, for example the mutual support mathematics and physics traditionally provided for each other can no longer be assumed. Many students also enter Higher Education from different routes, for example Access courses or with vocational qualifications, and as a result may not possess the solid mathematical background required for many courses.

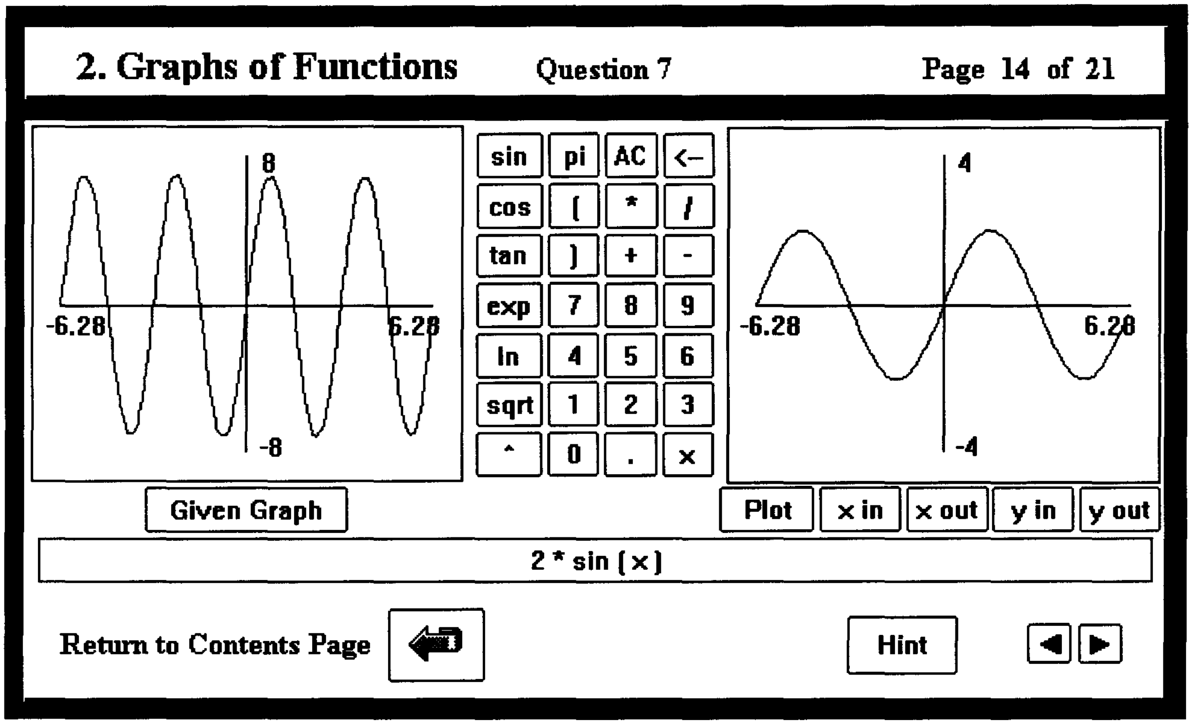
There is a distinct lack of motivation from students due to the fact that mathematics is considered by many to be a traditionally difficult subject. We must therefore strive toward the creation of alternative, more attractive approaches to the teaching and learning of mathematics, which will make studying mathematics at all levels an altogether more inviting prospect. It is the contention here that appropriate use of computer visualisation will assist this development.

4. Dynamic Approach to Functions

A survey of local teachers has shown that questions involving functions and their graphs are the most commonly quoted example at A-level of 'visual difficulties'. Many mathematics questions present a function, and the student is then required to plot its graph or describe in some way the key features of its graph. The question would be more 'visual', however, if the student were given a graph, and then asked what type of function the graph represents. It is this mathematical structure on which our initial interactive package is based. It is felt that this approach will lead to greater understanding of how students incorporate the concepts of functions through visualisation and yet give meaningful insight into improved imagery in studies of other mathematical topics. The interactive software, constructed using the generic authoring package TOOLBOOK, is briefly described.

The user is shown a graph, and then using a keypad will attempt to determine the correct mathematical expression for the function. Note that this is not possible directly on a graphics calculator. A graphics calculator can only produce a graph from a given function expression - it cannot produce the function of a given graph. This does not help in the acquisition of conceptual understanding.

Once the user has supplied a function, it can be plotted in a window adjacent to the given graph, and a comparison can be made. See the screen-shot below for an example:



This process can be repeated as many times as the user wishes. The package adopts a constructivist approach to learning, in that it invites the user to explore and examine various functions with a view to eventually arriving at a perfect match to the given graph (the user is also allowed to alter the scales of the axes by zooming in or out of either axis). The user can begin to appreciate the effect on the graph of altering various parts of the function. Once a graph has been plotted, the package supplies messages as to how near or far the user is from finding the correct function. The package contains several 'questions' which gradually increase in difficulty as the user progresses.

The package will assess how well users can interpret graphical representations of functions as opposed to their more commonly encountered algebraic forms. Users of this package should reinforce their conceptual understanding of functions, and at the same time enhance their ability to visualise.

The four main motivational factors, discussed by Lepper et al. [3], concerned with the maintenance of interest,

- the material needs to be challenging,
- the package must bolster the user's self-confidence,
- it must evoke curiosity,
- it must allow the user to be in control,

have all been incorporated in the design.

Now that the software has been prepared, experimental trials to attempt to quantify improved conceptual understanding via visualisation have to be planned. A written test has been prepared for evaluation purposes, which will assess a person's ability to switch between algebraic and pictorial representations of functions. The test will be given to upper-sixth mathematics students (or lowersixth students completing the end of the academic year) who have done a considerable amount of work with functions throughout the first year of their A-level course (they will not use the interactive package). Lower-sixth mathematics students, at the start of the academic year, who have done very little work with functions will use the interactive package, and then also sit the same written test (this is purely part of the supervised research project, and will not impinge on any planned teaching of this topic). The former can therefore act as the control group who have been taught 'functions' by traditional methods, and the latter can act as the experimental group who have learnt 'functions' via the interactive package. We can thus attempt to quantify the usefulness of the package in terms of 'learning', 'motivation', etc., compared with traditional teaching techniques.

The test will consist of 10 'mechanical' questions, Le. questions that cue the students to respond with an answer that involves the systematic application of basic knowledge or procedures (discussed by Galbraith and Haines [2]), and 10 'visualisation' questions, i.e. questions that require the students to switch between algebraic and pictorial representations of functions. We will thus be able to detect any differences in the ability in answering the two types of questions, and will gain some insight into the effect of the interactive software on student learning.

5. References

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