RESEARCHING GEOMETRIC THINKING IN OUT OF SCHOOL CONTEXTS

Zlatan Magajna
University of Leeds, University of Ljubljana
Zlatan.Magajna@uni-Ij.si

We consider the use of school maths in out-oi-school context. First, the notion of a context of an action is clarified and distinguished from some other notions of context. We apply Saxe's four parameter model to describe the context of an action and argue that the accomplishment of the mathematical goals that emerge during an activity is a necessary condition for the use of school maths in out-of schoolsettings. We wonder if this condition is also sufficient and claim that a CAD setting may be a suitable context for researching this question.

Problems related to context and its absence are, by tradition, a point of interest for mathematics educators. The advent of situated cognition introduced a new perspective for understanding and researching these problems. An analysis of recently published articles reveals at least 4 levels of analysis of context in maths education:

- the context of a word/utterance in a text, i.e. the linguistic context,
- the context of mathematics in culture,
- the context of a *mathematical concept* in an application, i.e. the context as is usually mentioned in maths applications in classroom settings,
- the context of a mathematical action in an activity (setting).

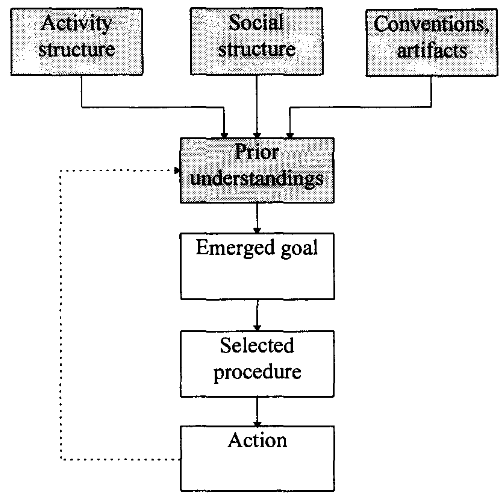
We shall focus only on the last level of analysis and try to relate mathematical actions to the activities in which they take place. The context of an action may be the school setting, a working situation, a playing activity or an everyday situation. There is much evidence that the mathematical actions that result from a task depend on the setting in which they occur, in particular we cannot expect that people use school learned methods and school learned ways of solving problems in out of school settings. This fact gives rise to the question: **when do people use school maths in out of school contexts?**

According to the situated cognition paradigm, the answer is: school maths is used only in schools. Even more: school-mathematics is used only in maths-classes, and there is evidence that pupils do mathematics in sciences 'differently' than in maths, at least certain topics (according to maths education folklore it takes in general three years for a student to integrate the knowledge of the same topic in different applications). Personally I do not share this radical opinion and I would like to find an answer to the next few questions:

- **Which factors significantly influence the mathematical actions undertaken to accomplish a task?**
- **Can mathematical actions be predicted from the task and the context?**
- **How do we describe and research a context (of actions)?**
- **What does all this have to do with school maths?**

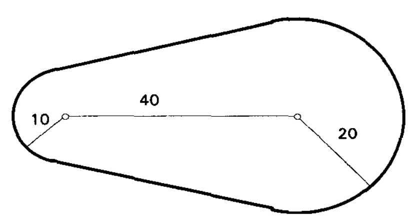
Saxe's 4-parameter model

A useful framework for researching the above questions is the four parameter model by G. Saxe (1991) as part of his methodology for investigating the influence of cultural practices in cognitive development. This model reasonably simplifies the intricate relationships between the participants' actions and the practice in which the tasks are performed. Furthermore, the model explicates the context-related parameters that most influence the way of doing mathematics in order to accomplish a task. Here is a slightly modified version of Saxe's diagram:



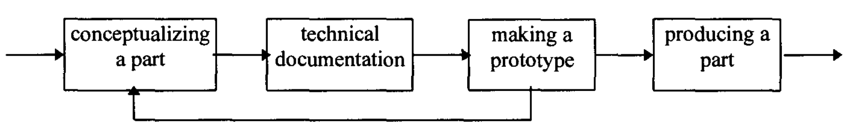
The basic idea of his approach that we shall use is that the four context related parameters define the mathematical goals that emerge during the execution of a task. (These emergent goals should not be confused with participant's goals in doing the task.) To accomplish mathematical goals the participants in an activity undertake suitable actions. Thus it is not the setting itself that 'prevents' the use of mathematical procedures learned in other settings, usually the goals that emerge in various settings are different and consequently different procedures are used. Naturally, with time the procedures used in a setting become associated and linked to the setting, but the linking mechanism was via emerged goals. Although Saxe applied his model to investigate the influence of cultural practices to maths knowledge on activities where only 'simple' arithmetic procedures were used I believe that his model can be proficiently applied to research the contexts in setting where more advanced mathematical thinking is required.

To get an idea how to use Saxe's model let us consider the task of drawing the following shape:



How such a task is executed obviously depends on the setting. A child with no knowledge of geometry may just make a freehand sketch of the drawing - in this case no mathematical goals would emerge. Let us describe an out-of-school context, for example, a shop-floor, using Saxe's model.

Activity **structure.** The task must be considered as it occurs in an activity cycle which consists of several functional phases. In our case a possible (simplistic) scheme of phases could be



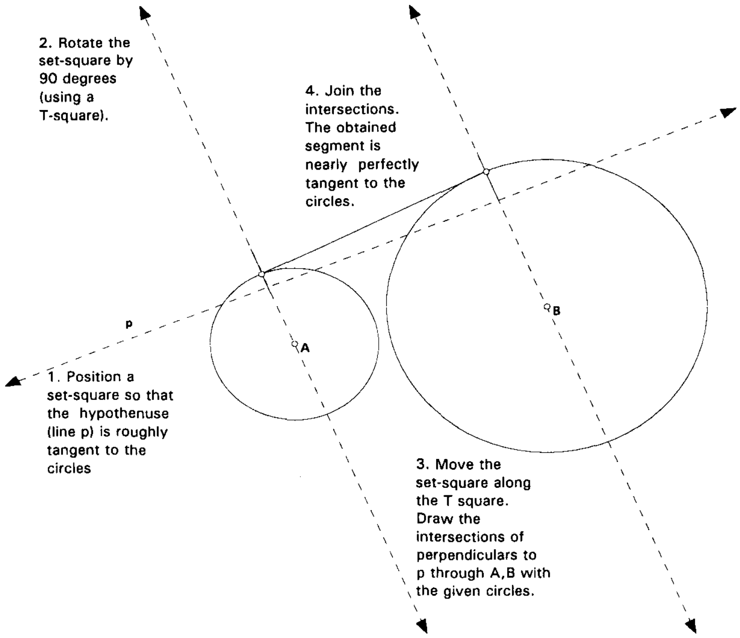
Apparently similar tasks in various phases may be executed for different purposes by the same or different participants in mathematically distinct ways.

Social structure. Mathematical knowledge may have important social dimensions (see e.g. Millroy, 1992). Whether a participant in a practice does or does not use a mathematical method often depends on the social connotations of the method. An engineer may refuse to draw a simple tangent because 'it is a draftsman's task' and a technician may not to do a mathematical calculation that is 'owned' by an engineer. Thus, in considering each functional block of activity one should consider the social organization of the participants involved in the activity, the social relations between the participants themselves as well as the social connotation of the commonly used methods.

Conventions and artifacts. The artifacts used, in our examples the drawing tools, obviously influence the mathematical goals that emerge during an activity. Hand-drafting the tangent segment involves several mathematical sub-goals in accordance to the selected construction procedure. In CAD drafting a single goal - to draw a tangent - is sufficient since the procedure is executed by a computer. The interpretation of a task often involves many context-specific conventions, the most remarkable are usually the notation and symbolization. In our case a draftsman understands that the segments are tangents to the circles because the endpoints of the linear segments are not depicted and dimensioned and it is a convention that in such cases the segments are tangents to the circles. Important conventions concerning maths related tasks comprise the required precision, allowance of errors, time constraints, credibility of results. In hand-drawing, for example, participants avoid making errors because it is time consuming to correct them, while on CAD the errors are easy to correct and are not particularly feared.

Prior understandings. A participant in a practice uses his/her knowledge in considering the mentioned parameters, in order to organize them and to relate the tasks to mathematical goals. In our example a draftsman's goal is not, for example, *to draw a tangent line to two given circles* or *to construct the points of tangency,* but *to join (quickly and) visually continuously the circular arcs with segments.* To accomplish such goal a draftsman will not use the school-learned procedure for constructing a tangent to two given circles, but, probably, an approximate method that, as it turns out, is in practice more accurate because of its simplicity. An example of such procedure is shown on the next page.

The school method for drawing tangents is correct but certainly not effective for the purposes of a draftsman. More precisely, the method is not well suited to the goals that emerged from the activity: the goal was to draw quickly a visually continuous joining segment not to construct the exact point of tangency. The school-learnt method (so important for understanding the property of tangents to circles) does not meet the needs of the emerged goal and thus other methods are preferred.



In considering the draftsman's act of joining two circular arcs with segments - as in many examples regarding the use of mathematics in out-of-school settings - we generally cannot resist from wondering: *Does this have anything to do with mathematics at all?* Indeed, the draftsman uses (more or less) the same tools as in school-setting, s/he is working on geometric objects and has even executed a very interesting approximate construction - but the draftsman's reasoning while executing the task was certainly not mathematical. Indeed, mathematical reasoning in work settings is often not particularly appreciated because it is time consuming and prone to errors. This is one reason why CAD systems used for drawing purposes tend to incorporate considerable mathematical knowledge in order to enable users to work efficiently without undesired errors.

Geometric thinking in out-of-school contexts

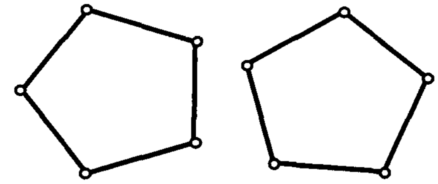
We have pointed out that the basic reason why school maths methods are usually not used in out-of school settings is that they do not effectively accomplish the mathematical goals that emerge from activities. In fact, the description of the context shows that the goals that emerge in classroom activities differ from mathematical goals that emerge in most other activities.

Assume now that in an out-of school practice *a school-learnt method (or the reasoning promoted in maths classes) is appropriate* for the realisation of an emerged goal? Will the participant use it or are there other condition to be met in order that school maths is used?

The question is, indeed, rather hypothetical, since it is not easy to find , say, a working setting where such goals naturally occur. I believe that CAD activity is a suitable context for researching such questions for several reasons:

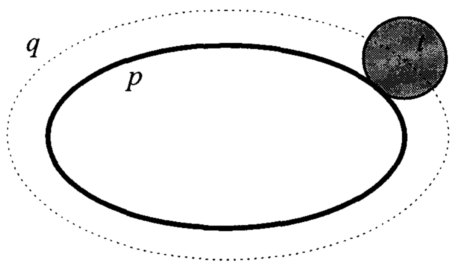
- Learning or doing CAD is not conceptually difficult (unlike learning in science class).
- Students are, in general, well motivated when working on CAD systems.
- In CAD activity intuitive reasoning and learning is common.
- In CAD activity tasks that can be effectively accomplished with school-math knowledge naturally occur.

As an example, assume that a CAD draftsman has to establish whether two apparently congruent (and 'nearly' regular) pentagons shown on a CAD display are exact copies of each other (the problem can be presented in a realistic and meaningful fashion). There are two essential ways how to solve

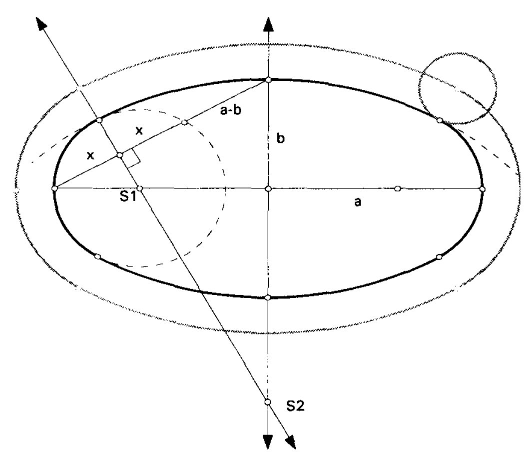


such problem, the CAD approach and the school-maths approach. If one sticks to CAD methods, one will try to find if it is possible to cover the second shape with the first shape (on CAD systems the shapes can be very effectively rotated and moved around). Obviously, this method is, in this case, very time consuming and inefficient. Another way, also common in CAD practice is to accurately measure the distances between selected points. One has to decide which distances and possibly which angles to measure and compare. Using the congruence theorems for triangles may significantly reduce the numbers of required measurements. Finally, one may completely switch from CAD and compare the areas or the perimeters ofthe two polygons (both functions are readily available on CAD systems).

From my experience I know that people in working practice occasionally revert to school mathematics.



A CNC programmer, for example, was faced with the task of cutting an elliptic shape (denoted by *p* in picture on the left) with a milling machine. To accomplish this task he had to calculate the path *(q)* along which the centre of the cutting tool (represented as a circle *t)* should move while cutting the elliptic shape *p*. The technician claimed that the offset curve *q* is also an ellipse, obtained from the ellipse *p* by incrementing the semi-axes by the radius of the tool. Although he was convinced of the correctness of his reasoning, he nevertheless asked me for confirmation (social factor!). He was certainly not happy to hear my explanation that the offset curve of an ellipse is not an ellipse and he didn't hide his surprise. After a while he came back smiling and announced that he had found a proof that the offset curve of an ellipse *is* an ellipse. Here is his argument: The ellipse consist of four circular arcs with common tangents at intersection points. By offsetting the four circular arcs we get four new circular arcs (with the same centres but bigger radii) and they touch each other smoothly: thus the offset of an ellipse is an ellipse. (Note that the technician had in mind a common practice of



approximating an ellipse with four circular arcs. The construction is shown on picture above.)

Such reasoning could be described as a mixture of a reminiscence of in-school learned methods and job related practices. Although incorrect in two deductions it was certainly a good step towards the solution of the problem of how to cut an approximately elliptical shape with a milling machine.

Conclusion

With the impact of technology in everyday and professional lives the common domain of school and work mathematics, consisting (at least apparently) of mostly commonly used procedures, is getting smaller. School mathematics is more and more concerned with the development of pupils' conceptual apparatus: the pupils in maths-classes are working 'on the edge of understanding' in a climate of uncertainty of newly learned knowledge, experimenting and making errors in order to reflect on them. In work settings people need a specific type of maths knowledge, often conceptually simple, errors are to be avoided and technology is widely used to solve mathematical problems. The goals that emerge in a school setting hardly resemble the ones that emerge in out-of school situations. It is thus reasonable to expect that the school learned way of doing maths is rarely used in informal situations and vice versa.

On the other hand some maths educators feel the urge to relate school-mathematics with out-of-school activities. An authentic presentation of such connections should take into account the context of mathematical actions and make explicit the mathematical goals that emerge from presented activities. A necessary condition for using school-learned mathematics in informal situations is the adequacy of emerged goal, but is it sufficient? Saxe's four parameter applied on CAD setting may help in revealing the answer.

References

Bishop, A. (1991), Mathematics Education in its Cultural Context, in Harris, M. (ed), *Schools, Mathematics and Work*, London, Falmer Press, 29-41.

Boaler, J. (1993), The Role of Context in the Mathematics Classroom: Do they Make Mathematics More "Real"?, *For the Learning of Mathematics* 13(2), 12-17.

Giesecke, F.E., Mitchell, A., Spencer, H.C., Hill, I.L., Dygdon, J.T., Novak, J.E. (1992), *Principles of Technical Drawing,* New York, Collier Macmillan International.

Lave, J. (1988), *Cognition in Practice*, Cambridge, CUP.

Lave, J. (1992), Word Problems: a Microcosm of Theories of Leaming, in Light, P. & Butterworth, G. (eds), *Context and Cognition: Ways of Learning and Knowing,* Harvester Wheatsheaf, New York, 74-92.

Masingila, 1.0., Davidenko,S., Prus- Wisniowska, E.(l996), Mathematics Learning and Practice in and out of School: A framework for Connecting These Experiences, *Educational Studies in Mathematics* **31,** 175-200.

Milroy, W.L. (1992), An Ethnographic Study of the Mathematical Ideas of a Group of Carpenters, *Journalfor Research in Mathematics Education,* Monograph no.5.

Saxe G.B. (1995), From the Fields to the Classroom: Studies in Mathematical Understanding, in Steffe, L.P., & Gale, 1., *Constructivism in Education*, Lawrence-Earlbaum Associates, Hillsdale, 287-311.

Saxe, G.B. (1991), *Culture and Cognitive Development: Studies in Mathematical Understanding,* Lawrence-Earlbaum Associates, Hillsdale.