MAKING CONNECTIONS THROUGH ACTIVE GRAPHING

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Abstract

Active graphing has been proposed as a spreadsheet-based pedagogic approach to support young children's construction of meaning for graphs, particularly as a tool for interpreting experiments. This report discusses the initial planning and early data analysis arising in a detailed study of the active graphing approach. In this study, we hope to gain a deeper understanding of the children's actions and the associated mental processes observed during active graphing.

Background

The research presented here is part of the Primary Laptop Project, which has now been running for over five years. The aims of the project are to develop a primary school environment in which teachers and children have high levels of access to portable computers, so that:

• we could study the effect on children's mathematical development,

• we could study other effects on the school, the teachers and the children, and

• student teachers might observe and appreciate the educational benefits of using computers.

The project is based in a primary school which caters for children from 5 to 11 years old. It is based on an 'immersion' model. For the period of time that they are involved in the project, each class is equipped with one machine for every 2 or 3 children. The children take full responsibility for their machine: they get it out, put it away, re-charge its battery when necessary, and decide who is taking it home. Although the main focus of the project is on mathematics, the computers are available for children to use across the whole curriculum. The planning and teaching of activities to develop the use of the computers in mathematics and science have been hared by teachers and researchers. With a few exceptions, all the teaching and research takes place in normal classroom situations.

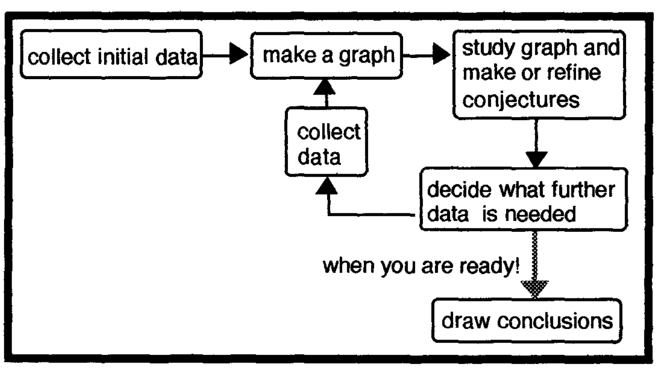


Fig 1: Active Graphing stressing interpreting a graph

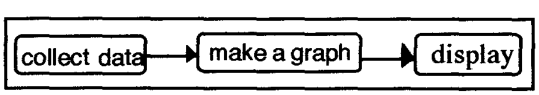


Fig 2: Passive Graphing evident when the presentational use of a graph is emphasised

Much of the early work of the project has focused on children's use of graphing within a spreadsheet environment. There is a considerable body of research evidence in both mathematics and science education, which suggests that children experience difficulties with graphing, particularly with the interpretation of graphs. Preliminary evidence from the Primary Laptop Project has led us to conjecture that these difficulties may be pedagogic rather than cognitive in origin (Ainley (1995). This has led to the development of a pedagogic approach we call 'Active Graphing' (Fig. 1), in contrast to the more passive approach (Fig. 2) normally experienced within primary schools (Pratt

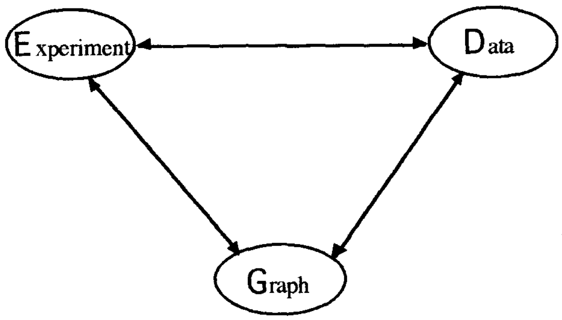


Fig. 3: The EDG Triangle

(1995)).

Our current project, funded by ESRC, is designed to explore further the features of active graphing which are particularly effective in supporting children's ability to interpret graphs showing relationships between two variables. In particular we are interested in exploring how children interact with the three modalities of the *experiment* or practical activity, the *data* which they tabulate on the spreadsheet, and the *graph.*, usually a scatter graph, drawn from this data. We encapsulate these different modalities as the EDG triangle (Fig. 3).

PlanningThroughout the Primary Laptop Project, collaborative planning between the teachers and researchers has been a feature of our work (Ainley & Pratt, 1995) It was therefore natural for us to begin by planning activities which fitted the broader themes proposed by the classroom teacher, Moira. The mathematics work for her class of eight and nine year old children during that half term was to be based on the theme of *paper.*

The discourse that was generated through this planning process was not only creative but fundamentally influential in our thinking. Although a range of activities was developed as a result of this planning, here we focus on just two which will serve to illustrate some ofthe main issues which were raised and which we feel are likely to inform subsequent stages of data analysis.

Two activities

Our previous studies of active graphing (Pratt, 1994, 1995a, 1995b) had given us a clear understanding of the attributes that make up an effective active graphing activity.

- Experimentally based

The activity must offer opportunities for practical experiment generating data, which can be appropriately represented in graphical form. To date, our most successful experiments have involved the use of non-categorical numerical data, represented as a scatter graph.

- Meaningful challenge

The activity must have a purpose which is meaningful to the child in the sense that the child can take ownership of the problem. At the same time, the graphical information must also be intrinsically useful and offer information that is not self-evident from the data or the experiment itself.

- Control of the independent variable

The child must have control over the independent variable and so be in a position of being able to choose what to do next in the experiment through deciding on a value for that variable in the next stage of the experiment (in contrast say to the common primary school activity where varous body measurements are taken in an attempt to find possible correlations. This otherwise worthy activity is not very effective from an active graphing perspective because the child's control is exerted through choosing a person and then measuring the two variables rather than through making an explicit choice about one variable and carrying out an experiment to find out the consequent value of the dependent variable).

- Iteration

The activity must allow for the experiment to be repeated many times and at each iteration for there to be the possibility of exploring the data in either numerical or graphical form. It must be possible for the child to draw conclusions from the data which will inform the next stage of the experiment.

The following two activities were designed with these attributes in mind and to fit naturally with the theme of paper.

In our planning, a number of potentially interesting issues were raised, and we wish to discuss these

Bridges	*Display Area*
The children were challenged to make a bridge by folding a piece of paper and to explore how strong the bridge was by identifying how much weight the bridge could carry before collapsing. In presenting this task, Moira introduced a heavy egg-shaped ornament so that, for some children, the challenge became one of designing a bridge which could hold the egg.	In this activity, the children were asked to design a rectangular display area, using a border which was 75cm long. The display area would contain many miniature pictures and so needed to have as large an area as possible. Moira suggested that the children pin the border to the floor so that they could spread the border out into a rectangular shape before measuring the length and width.

issues further using the above two activities as exemplars.

Epistemological characteristics

Two aspects of our planning discussions relate to the nature of the mathematical knowledge embodied (metaphorically) in the activities.

Mathematicalmodel

The display area activity contains an underlying mathematical model, relating the width to the length of the display area, $l = \frac{75 - 2w}{2}$. This mathematical model may be accessible, even in algebraic form, to some children of this age, especially with the support offered by a spreadsheet. More likely, some children will begin to calculate the length for any given width. In contrast, no such strategy is available in the bridges activity. Although one might suppose that the maximum weight held by the bridge increases as we' add more and more folds of paper, it is far from clear what the precise relationship between those two variable should look like.

Type of graph

The relationship between the two variables in each activity is quite different and this of course leads to contrasting graphical forms. The bridges activity is likely to lead to a roughly linear graph with positive gradient, whereas in the display area activity there is a quadratic relationship implying a parabolic graph.

Psychological characteristics

We wish now to discuss three contrasting psychological characteristics of the two activities.

A priori intuition

The children will inevitably bring a whole mass of intuitions gleaned from everyday and classroombased experiences. These informal ways of looking at phenomena will shape their interpretations of their experiments and the data. In our planning, we expected many children to recognise from the outset that bridges made with more folds would hold more weight. In contrast, we did not expect many children to appreciate initially that square-ish display areas would be larger in area.

Phenomenological Impact

We use this term to discuss the extent to which the actual experimentation will have immediate impact on the children's appreciation of the data. For example, we expected in the bridges activity that the children would gain a feel from their experiments that bridges made with more folds were holding more weight. High phenomenological impact combined with strong a priori intuitions imbues the bridges experiment with the potential for the E and D vertices of the **EDG** triangle to offer strong support for children's interpretation of G.

In contrast, we expected the display area activity to have low phenomenological impact. The carrying out of the experiment in itself is likely to make little impression since it is difficult to distinguish between small and large areas merely by inspection.

Metaphoric Resonance

This term describes the possible connections that the child may make between features ofthe experiment and qualitative aspects of the graphical image or numerical data. For example, in the bridges activity, we might expect some children to link the growing graph to the increasingly tall pile of weights placed on the paper bridge. Or, perhaps the children would read the graph right to left because they expect the graph to go *down* as do the weights when the bridge collapses. In the case of the display area, we could see little potential for metaphoric resonance.

The Structure and Content of Data Collection

With Moira's collaboration we designed and practised four Activities- Bridges and Display Area are the first two - each to be used in the classroom for four consecutive days in a week. The activity was introduced to the class by Moira on Monday morning. Eight children (2 groups x 2 pairs) were selected for close observation and interviewing by the researcher carrying out the data collection. Selection was not on the basis of mathematical ability but aimed at covering various degrees of openness and communicative skills. Each group worked on the activity for two days while the rest of the class including the other group - worked on other non-mathematical tasks.

The sessions with the groups were tape-recorded as was the closing session with the whole class on Thursday afternoon. Data collection has just been completed: the data consist of approximately 30 hours of recorded material. Significant incidents related to active graphing are currently being selected for transcription and analysis. In this paper we provide a small sample of the data collected while the second activity, Display Area, was used in Moira's class.

One incident with Laura and Daniel

During the two days, the pair engage in a number of acts which we now begin to see as specific aspects of active graphing: noticing properties regarding the trend or the shape of the emerging graph (Trend-Spotting T S, Shape-Spotting S S), identifying maximum values of the graphed function (MaxSpotting MS), concentrating experimental activity on trying to match or surpass these values (MaxZooming MZ), changing initial preconceptions about the experiment CP, Spotting Inaccuracies or Redundancies S I in their trials, Testing Hypotheses TH. based on their observations of the graph, the data or the trials of the experiment and Adopting Accurate/Formulaic Practices AFP as a result of these observations. All these acts are carried out as closely intertwined interactions between the three aspects of the EDG triangle. During data analysis it is intended that a wide variety of the observed aspects of acti ve graphing as well as the interactions between the three aspects of the EDG triangle will be described.

Summary of Laura and Daniel's Work

Construction of rectangles starts. It is slightly delayed by the children's difficulty to make the rectangles accurately. The pair open a Spreadsheet on their computer and start entering values for the width, the length and the area of the rectangles in three columns. The entries in the area column are calculated as the product of the entries in the width and length columns (AFP). After a few trials of the experiment they make their first xy-Scattergraph (Fig. L-D 1).

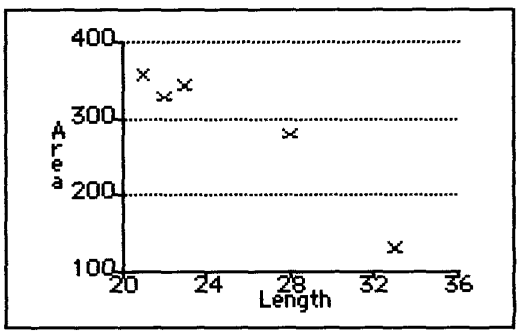


Fig. 4: L - D 1

While observing this, Laura says that her prediction about the area is that it 'will become bigger first and then smaller' (TS). During the trials, the two children notice that 'short and fat' rectangles 'make more area' than 'long and thin' ones. The word 'square' comes up for the first time and it is followed by a debate between them about whether a square is a rectangle or not. The debate remains inconclusive as Daniel proposes making the rectangle of zero length which they both agree that it will make zero area.

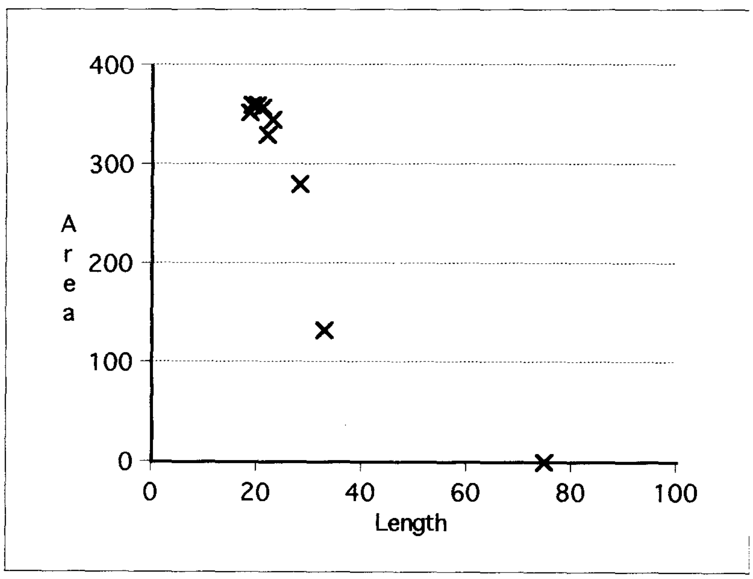
Day 2 begins with the pair reminding themselves of what the graph, made in Day 1, looked like. Laura says that this activity is about 'looking at the trend of the graph' and also 'looking at the highest' so far. The two children describe what they expect to see on the graph: a 'hill' or a 'mountain' (TS/S S). While looking at the columns of data, Laura notices that as width becomes bigger, length becomes smaller. For the rest of Day 2, Laura is in charge of entering the data on the Spreadsheet and Daniel is making the rectangles.

Laura is constantly looking at the data and the graph in order to identify the highest area achieved so far (MS). Her suggestions to Daniel about which rectangles to make next are based on her intention to match or surpass the highest score achieved so far. On the way some of her initial preconceptions about the task are changed (such as that larger length implies larger area; or that, because the given perimeter of the rectangle is 75 cm, the largest area must be 75 cm^2). She also brings to Daniel's attention redundancies (repetitions of the same dimensions of the rectangle or reversals of width and length that lead to the same area) or inaccuracies (she notices it is impossible to make both the 21x17 AND 20x17 rectangles and she asks Daniel to choose one of the two) of their experiment.

While entering the result for the 17x21 rectangle Laura exclaims that 'the final result will be a square!' (TH). Subsequently, and in order to verify or falsify Laura's exclamation, the pair work in the zone of 18 - 20 cm of length. After a long series of attempts to make a square of lengths of 18, 19 and 20 cm fail and after changing her mind that 'it has to be a whole number because all squares you see are whole numbers', Laura, who firmly believes that 'But there must be one number, ONE NUMBER that makes a square!'. 'somewhere in the twenties', finds out that 'Oh it can be a half or a quarter!!!' (MZ).

Away from the computer and totally focusing on the construction, the pair decides to 'go through stages until we make one' and to 'test all of them out'. Laura is determined that the length they are looking for is 'landing on a certain number' and after several attempts she proposes they 'divide (the75 cm] by four' and then make the square (MZ).

When the square of side 18.75 cm is finally constructed, the pair return to the computer (Fig. L - D 2) where, to their immense surprise, the

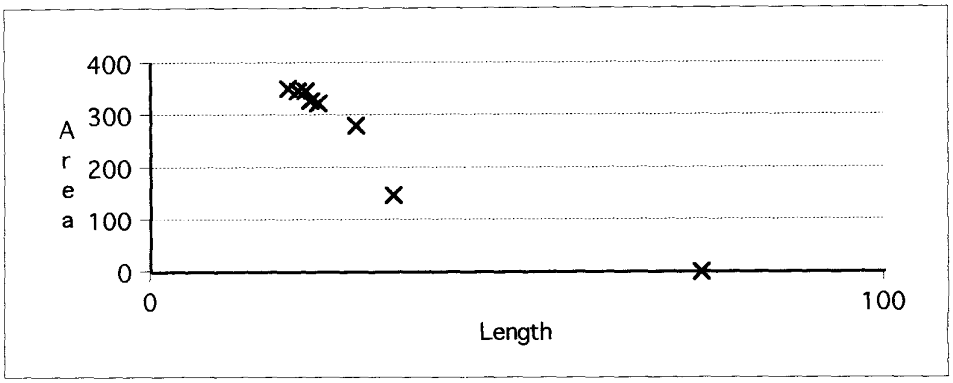


Width	Length	Area
15	23	345
10	28	280
4	33	132
15	22	330
17	21	357
0	75	0
18	20	360
19	19	361
18.75	18.75	351.5625

Fig 5 : L - D 2

area 351.5625 is not the largest among their data.

Their initial disillusionment is changed by the researcher who brings their attention to the inaccuracies of some of their measurements (e.g. the 18x20 rectangle whose perimeter adds up to 76 cm and not the required 75 cm): as a result the pair retype their data accommodating more accurate measurements based on the relationship between length and width. In the revised columns (Fig. L - D 3) the square is the rectangle of largest area *(we may refer to this as 'adopting accurate practices')* .



Width	Length	Area
14	23	322
10	28	280
4.5	33	148.5
15	22	330
16.5	21	346.5
0	75	0
17.5	20	350
18.75	18.75	351.5625

Fig 6: L - D 3

The two children seem to be convinced that the square is the largest area that can be achieved: in the closing session on the last day, where the pairs present their work to the rest of the class, Laura demonstrates L - D 3 and says, while pointing at the top of the graph, that they expected the graph to 'go up and then down' but they only had time to make this part. She then describes how they gradually realised that the answer was 'in the twenties', how it was not related to a whole number but to a 'half or a quarter' and how they finally managed to reach the 18.75 cm of length.

The above summary indicates the approach we adopting in trying to categorise processes which are observed during active graphing. We hope to reach an understanding of how these processes interrelate with each other and with the EDG triangle.

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