THE ROLE OF NUMBER SENSE IN CHILDREN'S ESTIMATING ABILITY

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This paper presented findings of a study looking at the comparative and combined effects of age and number-sense on children 's ability to estimate measures. While evidence was found for a developmental effect of age on children 's number-sense, no such effect was found for the ability to estimate either length or area. However, childrens' ability to use and perceive number relations, together with an understanding of the relative magnitudes of larger numbers, were found to have a significant influence on their ability to estimate area.

Introduction

The influence of numerical understanding on the ability to estimate has been assumed, observed or remarked upon anecdotally in most studies of estimating ability (eg.Carpenter et aI, 1976; Siegel et aI, 1982; Forrester et al, 1990). However, while knowledge of any domain is clearly an important prerequisite for effective estimation within that domain (Brown & Siegler, 1993), the precise nature of the relationship between number knowledge and estimation remains unclear.

Recent discussions of numerical ability have emphasised the role of 'number-sense'; a term used to encapsulate an holistic concept of quantitative intuition, or a 'feel' for numbers and their interrelationships (Sowder, 1992; Markovits & Sowder, 1994). Conceptually, number-sense has been taken to include the recognition of the relative magnitudes of numbers, the effects of operating on numbers, and the development of benchmark referents for quantities and measures (Sowder, ibid). It has been operationally defined in such terms as the ability to use numbers *flexibly* when computing, estimating, judging number magnitude, or judging the reasonableness of results; the ability to *move easily* between different number representations; and the ability to *relate* numbers, symbols and operations (Markovits and Sowder, ibid). The emphasis here is of a non-reductionistic kind, focussing upon the creative *use* and *perception* of numerical relationships, rather than accumulated knowledge or isolated skills.

By far the most research on estimation has focussed on computational estimation, as opposed to the estimation of measures. However, many activities frequently include elements of each, and both have been noted as sharing certain mathematical processes in common. For example, both necessitate some form of decomposition / recomposition, and/or the use of benchmark values (Bright, 1976; Siegel et aI, ibid), in addition to such processes as counting, visualising, and approximating (Ainley, 1991). Following previous work (Forrester et aI, 1990; Forrester & Shire,1994), the present study aimed to clarify the developmental relationship between number sense and estimation of measures in young children. Specifically, our aims were to identify (a) the developmental effects of age on both number-sense and estimating ability, and (b) the relative influence of age and number-sense on children's ability to estimate measures.

Method

A total of 62 primary school children (38 m; 24 f) ranging in age from 6-11 yrs, completed all tasks over a period of seven months (two school terms). All tasks were produced using Macromedia

Director multimedia authoring software, and presented on a small laptop computer. Data were recorded directly by the computer, in a form suitable for subsequent analysis.

Assessment of Number-Sense

Number sense was assessed using three tasks, each focussing on one aspect of number sense (although necessarily including certain elements of the others).

(i) Mental Computation

This task required each child to solve arithmetic problems, randomly generated within specified constraints corresponding to a given level of difficulty, and presented in the form of thought bubbles emerging from an animated head. The successive difficulty of each problem depended upon a running score, which itself increased or decreased according to the child's responses. This feedback loop thus caused the score to eventually level out at a number taken to be a measure of the child's mental computational ability. A return to the same score three times was taken as the criterion for stopping the assessment. Rexibility was built into the program such that the assessor was free to choose an appropriate entry level, or to jump between levels at any point during the assessment. working level of ability.

(ii) Understanding of Relative Number Magnitude

Children were assessed on their understanding of the relative magnitude of numbers firstly from zero to one hundred, and secondly from zero to one thousand. The display presented a horizontal line from which ten 'strings' were vertically suspended, together with ten 'oranges' marked with selected numbers between, and including, zero and a hundred, or zero and a thousand. The task consisted firstly, in placing the oranges on the strings in correct numeric order from left to right, and secondly, in moving the strings horizontally such that the spaces between them reflected the relative magnitude of their corresponding numbers. Children were given a brief demonstration, and then asked to "move the strings along the line, so that the spaces between the strings show how much larger, or how much smaller, each number is than the other numbers". A measure of overall performance, termed the Relative Number Magnitude Index (RNMI), was then calculated on the basis of comparisons between the actual and expected positionings of each number relative to the others (unfortunately, restrictions on space prevent further elaboration here).

(Hi) Understanding of Number Relations

As a capacity to learn, or to perceive and use number relationships, is implicit in the idea of number-sense, this assessment drew on Vygotsky's notion of a 'zone of proximal development' (ZPD) (Vygotsky, 1978) in an attempt to take this into account. Conceptually, the ZPD refers to the 'distance' between what a child is able to do independently in some domain and what s/he is able to do in collaboration with a more experienced other. The former may be regarded as the 'actual' developmental level, and the latter as the 'potential' developmental level. In practical terms, a ZPD assessment typically makes use of some form of graded-prompt procedure (Brown & Ferrara, 1985), in which a child's learning is assessed in terms of the number of standardised prompts needed to complete a task. Children's understanding of number relations was here assessed in a similar manner.

Each child was first presented with the sum" 18 + 36", and asked to think of as many ways as they could of how they might solve it 'in their head'. There were three target strategies; an 'add on / take off' strategy, one decompositionlrecomposition strategy based on tens and units, and another based on factors. Where it was clear that the child was simply relying on a mental image of a written algorithm, this was recorded seperately. All other spontaneous strategies were recorded under one global category (although in practice, the only other strategy suggested was 'counting on').

If children suggested, and successfully used, a target strategy spontaneously without prompting, they scored four points. After all unprompted suggestions had been made, the assessor began the prompt sequence for each of the three target strategies in turn. (For reasons of space, the prompt schedule has not been included in this report). Whenever a child was able to use a prompt to successfully solve - and/or explain how to solve - the problem, the prompt sequence for that strategy was stopped and the corresponding score recorded. The fifth prompt in each sequence amounted to telling the child how to do it, and scored zero points. The total score across all three target strategies was taken as the overall measure of task performance on that session.

Children then repeated the assessment with a new problem, "24 + 48", and the overall mean score across both sessions taken as a measure of their ability to use and perceive number relations.

Assessment of Estimating Ability

Children were assessed on their estimation of both length and area within each of two distinct contextual frames. One of these presented the task within a stereotypical mathematics 'textbook' format; the other within a 'story' format. The story itself was designed so as to appeal to children across a wide age range, and concerned the plight of some ladybirds amidst a deluge

(i) **Length**

The 'story' task was preceded by a short animated sequence (including sound), during which a twig floated downstream, coming to rest at a leaf on which several ladybirds were stranded. The ladybirds then walked onto the twig one by one, forming a line from one end of the twig to the other, after which the twig floated away again to safety. Children then completed each of six trials, during which twigs of various lengths floated downstream towards the ladybirds, whose size also varied from one trial to the next. Each trial required the children to estimate how many ladybirds of the given size would be able to fit along the twig, and so escape to safety. Each trial of the corresponding 'textbook' task simply required children to estimate the length of a horizontal line in the centre of the screen, using a shorter line at the bottom of the screen as a unit measure

(ii) **Area**

As with length, the area 'story' task was again preceded by a short animated sequence, here consisting of ladybirds crawling around in a puddle during a heavy rainfall while attempting to climb onto a floating leaf. Following this, children then completed six trials in which they were presented with leaves of various sizes, and required to estimate how many ladybirds of a given size would be able to fit onto each leaf 'leaving as little gaps as possible'. Each trial of the corresponding 'textbook' task required children to estimate the area of a rectangle in the centre of the screen, using a smaller rectangle as a unit measure .

. All unit target ratios were the same across both 'story' and 'textbook' conditions in each domain of estimation, and the order of presentation of trials randomised across subjects.

Results

Age-Related Differences

No significant differences in estimating ability were found across age groups. However, children were significantly better overall at estimating length than they were area ($F = 153.9$, df 1; $P < 0.001$). 9-11 yr olds were significantly better at mental addition / subtraction than 6-8 yr olds ($F = 8.9$, df 5; $P < 0.001$), while 10-11 yr olds were significantly better at multiplication / division than 6-9 yr olds ($F = 14.2$, df 5; $P < 0.001$). No significant differences in mental computation ability were found within these wider age groups.

In general, understanding of relative number magnitude also improved with age ($F = 3.35$, df 5; $P = 0.01$), while children's understanding of the relative magnitude of numbers to one hundred was better overall than that of numbers to one thousand ($F = 99.88$, df 1; $P < 0.00 1$).

While all age groups improved significantly across sessions on the ZPD task ($F = 64.05$, df 1; $P < 0.001$), 9-11 yr olds' use and perception of number relations was significantly better than that of 6-8 yr olds age ($F = 7.22$, df 5; $P < 0.001$). Again, there were no differences within these wider age groups.

In summary, then, whereas children's number-sense did show improvement with age, their estimating ability did not.

Number Sense and Estimating Ability

No correlation was found between children's estimation of length and any variable except their estimation of area. However, estimation of area correlated highly with all three measures of number-sense. Standard regression analysis showed 22% of the observed variation in children's estimation of area to be accounted for in terms of age, mental computation, understanding of number magnitude, and use and perception of number relations ($R2 = 0.22$; $F = 4.04$, df 4; $P < 0.01$). Examination of part correlations suggested the greatest influence to be due to mental computation. (Indeed, a stepwise regression accounted for 19% of the variation in estimation ability in terms of mental computation alone ($R2 = 0.19$; $F = 13.45$, df 1; $P < 0.001$)).

As would be expected by definition, all three measures of number-sense were themselves highly intercorrelated. However, whereas children's mental computation correlated highly with both their understanding of the relative magnitude of numbers to one thousand, and their use and perception of number relations, the latter two variables were less highly correlated with each other. This suggested that understanding of number magnitude and number relations might usefully be considered sub-components of mental computation ability. On omitting mental computation from the analysis, subsequent standard regression showed 19% of the variation in childrens' estimating ability to be accounted for in terms of their understanding of relative number magnitude to one thousand, and their use and perception of number relations ($R2 = 0.19$; $F = 3.29$, df 4; $P < 0.05$) (ie.comparable to that accounted for by mental computation alone). Examination of part correlations suggested the greatest influence to be due to their use and perception of number relations, and secondarily their understanding of magnitude. (Stepwise regression found 15% of the variation in estimating ability to be accountable for in terms of use and perception of number relations alone ($R2 = 0.15$; $F = 10.82$, df 1; $P < 0.01$».

Hence, whereas childrens' ability to use and perceive number relations and, secondarily, their understanding of relative number magnitude, appeared to influence their ability to estimate area, no such influence was found on their ability to estimate length.

Discussion

These findings offer some support for the complementary roles of benchmark knowledge (relative magnitude of numbers) and decomposition-recomposition strategies (use and perception of number relations) as aspects of number-sense influencing children's ability to estimate measures (specifically, area). However, the fact that number-sense accounts for only 19% of the observed variation in estimating ability suggests other factors must also be involved. This is further implicated by the somewhat anomolous finding that while number-sense improves with age and estimating ability improves with number-sense, estimating ability itself does not improve with age.

Given that the domain of estimation under investigation is spatial, it would seem reasonable to hypothesise a possible role for spatial imagery ability here - specifically, the ability to mentally scan and manipulate such images. Existing research in this area suggests that younger children are relatively poor at this (Kosslyn et al, 1990), and we are currently investigating the comparative roles of imagery ability and number-sense in influencing children's estimation of volume. However, our previous research (Forrester & Pike, in press) suggests that children's prior experience of learning to estimate - and what that actually means in terms of social practice in a situated classroom context may be another major, if not overriding, influence. Specifically, what children understand by 'estimating' may be highly variable and context-specific, such that their 'ability' in this area tends to be defined and embedded within particular kinds of classroom activity and relationships, rather than be seen as a general cognitive skill (in contrast, say, to 'mental arithmetic'). One consequence of this is that children tend neither to be taught, nor to invent, general strategies for making estimates.

The study also highlights a need for testing methodologies and appraisal methods that take an ability to learn into account, rather than 'controlling' for it. The ZPD graded-prompt technique used in the present study represents one possible step in that direction, and in a later study we plan to use a similar methodology for assessing estimating ability itself. This will then allow investigation of whether children's ability to learn in one domain (eg.number) may be related to their ability to learn to estimate. Such an approach both acknowledges children's prior experience (or lack of experience) of estimating, and avoids the unnecessary and misleading conceptualisation of estimation as a static 'abili ty'.

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