

AN ANALYSIS OF STUDENTS TALKING ABOUT 'RE-LEARNING' ALGEBRA: FROM INDIVIDUAL COGNITION TO SOCIAL PRACTICE

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Abstract

In this paper we report on a study with the aim of investigating how a focus on language and meaning can assist students in reconstructing algebraic knowledge. The project is set in the context of ongoing work with students in Higher Education who need to develop their understanding of algebra if they are to make substantial progress within their undergraduate studies. The project is based upon a belief that students' difficulties with algebra are language-related. We have collected extensive data by means of videotaped sessions involving the students talking about their own understandings of algebra. The students involved were drawn from courses in other education and engineering. This paper presents a detailed analysis of the responses of one student and discusses the ways in which this shaped our attention as researchers. It looks at all the data from the perspective of individual cognition towards one informed by social practice theory.

Introduction

The Re-Learning Algebra project grew out of the difficulties many students have with algebra which have been observed in the course of working in the arena of Academic Maths Support at Sheffield Hallam University. These students have considerable prior experience with algebra and many have undergone years of drill and practice. They have encountered algebra as both an abstract topic in its own right and also within various contexts. Therefore any additional help offered to such students clearly needed to take account of previous experience but also needed to have a different emphasis. An approach which was seen to be successful in practice involved encouraging interaction using group activities in which the students could share their understanding and experience. The activities also addressed the use and development of algebraic language and have previously been reported on in Elliott and Johnson (1995).

Related Literature

The literature cited in this section of the paper helped to formulate our thinking and has informed our discussions during the course of this study. There has been much research on algebra at the school level e.g. Lesley Booth (1984), including the studies concerned with school children's errors in algebra and an analysis of these errors leading to a categorisation of types of meaning associated with algebraic notation. Anna Sfard and Liora Linchevski (1994) developed their theory of *reflexification* according to which there is an inherent process-object duality in the majority of mathematical concepts. They propose that the development of algebraic thinking is accomplished by means of a sequence of ever more advanced transitions from the operational to the structural. In particular they consider two especially crucial transitions: that from the purely operational algebra to

the structural algebra 'of a fixed value' i.e. an unknown and then from there to the functional algebra of a variable. Carolyn Kieran (1989) emphasises the recognition and use of structure as a major area of difficulty in algebra. She reports that for students, who tend to view the right hand side as the answer, 'the equation is simply not seen as a balance between right and left sides nor as a structure that is operated on symmetrically'. Carolyn Kieran and Nicholas Herscovics (1994) propose the existence of a *cognitive gap* between arithmetic and algebra which can be characterised as 'the student's inability to operate spontaneously with or on the unknown'.

We found a different emphasis in the work of Abraham Arcavi (1994) which seemed closer to our initial starting point for this study. He outlines what he terms as 'symbol sense' and which he compares to the notion of 'number sense' which has received far more attention. His work is about describing and discussing behaviours and not about defining and describing research on students' cognition and ways of learning. There is an emphasis on sense-making in mathematics and on recognising meaning.

Theoretical framework

Given the initial aim of this study, which was to investigate how a focus on language and meaning can assist students in reconstructing algebraic knowledge, we have sought to develop a theoretical framework which takes account of this emphasis on language and meaning.

A key influence has been the work of Lev Vygotsky (1962), underpinning which is a central assumption that socio-cultural factors are essential in the development of mind. Intellectual development is seen in terms of meaning making, memory, attention, thinking, perception and consciousness which evolves from the interpersonal to the intrapersonal.

In exploring the notions of sense and meaning further some useful ideas were drawn from the field of activity theory. In particular Erik Schultz (1994) offers some interpretations of sense and meaning, which we found to be helpful in interpreting some of our data. He proposes that the purpose or intention of a cultural product is the meaning and further that meaning is a kind of 'cultural intention' in a supra-individual fashion. Sense is the interpretation one makes of the meaning. He argues further that language is a special kind of cultural product. We also found the work on activity theory of Kathryn Crawford (1996) to be relevant. She highlights how activity denotes personal (or group) involvement, intent and commitment that is not reflected in the usual meanings of the word in English. In building upon Vygotsky's work, Leont'ev, Davydov and others made clear distinctions between conscious actions and relatively unconscious and automated operations. Operations are seen as habits and automated procedures that are carried out without conscious intellectual effort.

Methodology

Data was collected by means of the video recording of a series of one-hour sessions with four groups of students during March 1995. The students involved were drawn from courses in Education Engineering. The groups had two or three sessions each.

A series of tasks was devised which were designed to get the students talking together about their understanding of algebra. For example the first activity involved 'Algebraic Pairs'. In this activity

each group of two students is given a set of cards with a pair of algebraic expressions on each. The task is to decide if the two expressions are always, sometimes or never equal. Another activity was to ask them to explain what they understood by mathematical words such as expression, equation, function, variable etc. The sessions were carried out in a small TV studio.

The initial data analysis involved the three researchers simply viewing the video tapes and discussing reactions and questions arising. Following the tape transcription this process was repeated with the transcripts. Our discussions were further informed by our ongoing reading. We also held two internal university research seminars during this period.

In this paper we have chosen one particular section of the transcript which we found to be particularly rich but also very challenging to us to make sense of in terms of the starting point of our study i.e. how a focus on language and meaning can assist students in reconstructing their algebraic knowledge.

Data Analysis and Discussion

This particular section of the interaction took place at the end of the first session with the 2 Year BEd students. They had been working on the Algebraic Pairs activities for the first part of the session and then had spent the latter part in a discussion of mathematical terms such as expression, equation, function, variable etc. As the session was almost complete, the researcher provided the opportunity for any questions, reactions or general discussion. The result was an extensive and articulate series of responses from one student in particular - Anthony (AG) Anthony is a mature student who had previously worked in industry as an engineer.

1. **BH** OK, I was going to think about further activity but seeing as we've only five minutes left, I
2. think we'll end. Unless, are there any particular things that struck you as we've been talking, that
3. you want to return to, words which conjure up ...
4. **AG** It's obvious as we start talking about maths, we start talking about functions, some people
5. have got a clearer view; that my image I realise now, when I'm teaching, I tend to opt for, I like
6. to see it as, that $y = \text{some function}$, it could be $a=3b$ plus something. I keep returning to $y =$
7. some function of x and if I saw it in a textbook for example that $2(x+3)$ my automatic reaction
8. would be to write $y=2(x+3)$ before I give it to the children to do. $y=2(x+3)$
9. **BH** What would you be thinking of asking them to do next?)
10. **AG** I'd be asking them to multiply the brackets out to give me a $y=2x+6$ or asking them to
11. substitute a value of x and tell me what y is because that on its own as a function - $2x+3$.
12. suppose to me it is just floating about in mid-air with no relationship to anything it's totally
13. intangible, what is it? what's it for? So if I ask them to multiply that bracket out I got $2x+ 3$
14. before, now I've got $2x+6$, still doesn't lead to anything, doesn't mean anything doesn't tell me
15. what it's from or where it's from - so my automatic reaction is to put the $y=$ in
16. Otherwise you've got that floating about and that is a function, then you've got function. To me. what is a function,
17. where does it come from, where does it come from?
18. **BH** You'd be happy to relate it to y . What would that mean then for you?)
19. **AG** There's a missing number y and a missing number x and if we put any value in for y , or any

20. value for x ... If we can find a value for y then we can find a value for x and if you get into a
 21. quadratic there'd be two answers for y, so actually you're using something to solve a problem.
 22. **BH** Just taking that, say it was $y=2x$ squared plus 6 times something ... You said two values. 23. AG
 Again, as soon as you get an x squared, I tend to think that it's probably going to be two 24. answers.
 Depends on ...

25. AG I'm coming from a realistic point of view in that I've got a specific problem of trying to find
 26. out what this value of y is and in doing that I've made an equation in order to solve my problem
 27. and in trying to solve my problem I might find that there are two values of the \.

28. **BH** Say we had that? What about if I said y was minus 10)

29. AG Minus 10? Then there might not be a solution to it no real solution. No solution to my real
 30. world. This idea of no real solutions - you've gone into a hypothetical world You've gone out of
 a

31. real life situation. From my experience, in my situation, you've gone out of a real life situation,
 32. you're going back to a hypothetical situation. You're going right full back in circles to functions,
 33. that's something hypothetical - that's floating about, not related to anything or solving anything.

It's

34. not come from any real life situation, it's just a function, it's not related to anything else I think 35. that's
 why I have difficulty in seeing where it's coming from.

An initial analysis of this section suggested a number of links with the background literature and theoretical framework previously outlined. In order to help the reader make sense of the transcript, it is worth emphasising at the outset that Anthony does not distinguish between the terms function and expression. In fact he refers to $2(x+3)$ as a function rather than as an expression at line 7. In relation to activity theory there are a number of references to a lack of purpose when dealing with functions. For example at line 13, Anthony asks 'what is it) what's it for)' and at line 14 says that it 'still doesn't lead to anything' and goes further to say that it 'doesn't mean anything' This statement fits with Erik Schultz's interpretation of meaning as the 'purpose or intention' of the cultural product' which in this case is the word 'function'. Anthony's description also suggests that he is working operationally for much of the time e.g. at lines 7/8. he says that 'my automatic reaction would be to write $y=(2x+3)$ ' and also at line 15 he says that 'my automatic reaction would be to put the y= in'. His comments also suggest a lack of appreciation of the structural properties of equations e.g. at lines 10/11 he would 'be asking them (the children) to ... substitute a value of x and tell me what y is' This suggests a view, consistent with the work of Carolyn Kieran, of 'the right hand side as the answer'. His comments at line 19 'There's a missing number y and a missing number x' suggest that he has not made the transition, in Anna Sfard's terms, from the 'structural' algebra of 'a fixed value' to the 'functional' value of a 'variable'. It seems from Anthony's comments that he sees the purpose of an equation as being to find a missing number and not to express a relationship. In Carolyn Kieran's terms, the equality relationship is not fully recognised i.e. the equation as a balance between right and left hand sides and as a structure to be operated on symmetrically

To an extent these observations are typical of many students although they were surprising to the researchers, as Anthony was seen to be a mathematically capable, though not strong, student. However much of what Anthony had to say was left untouched by this analysis and we were left with a sense of the inadequacy of the various theoretical frames, through which we had viewed our data, to account adequately for what Anthony had to say. It seemed that there was evidence of resistance to 're-learn' algebra on Anthony's part and much that was being said about his sense of identity and also his view of the nature of mathematics. None of this seemed to have been addressed in our first readings of the data. As a result of wider discussions with colleagues we decided to look to social practice theory for a 'wide(r) angle lens' (Robert Dengate and Stephen Lerman, 1995) through which to view our data. In particular we turned to the work of Jean Lave and Etienne Wenger (1991) and that of Jean Lave (1996).

Jean Lave and Etienne Wenger stress the essentially social character of learning and propose learning to be an aspect of a process of participation in socially situated communities of practice. They discuss the notion of Legitimate Peripheral Participation (LPP) which describes the particular mode of engagement of a learner in a new community of practice, whose level of participation is at first legitimately peripheral in the practice of the expert. The move from peripheral participation to full participation is seen as a dynamic process, characterised by changing levels of participation. Writing in 1996, Jean Lave describes the direction of movement as a *le/os* and gives the example of 'becoming a respected, practising participant among other tailors or lawyers, becoming so imbued with the practice that masters become part of the everyday life of the Alley or the Hlosque for other participants and others in their turn become part of their practice'. She proposes that this might form the basis of 'a reasonable definition of what it means to construct identities in practice'.

Returning to the analysis of the transcript, it seems that there is considerable resistance on Anthony's part to reconstruct his view of algebra. His view of a function is that 'it is totally intangible' (I 13) and 'with no relation to anything' (113). It is 'floating about in mid-air' (I 12), without meaning e.g. 'what is it?' or purpose 'what's it for?' (113). It seems that Anthony's view of mathematics is only meaningful if 'you're using something to solve a problem' (121). Having a problem to solve is real e.g. 'I'm coming from a realistic point of view' (125) and equations are simply tools to solve 'my problem' (127) e.g. 'I've made an equation to solve my problem' (125). In formulating his views on the nature of mathematics, Anthony also seems to be saying significant things about his own sense of identity. His background is that of an engineer working in industry over many years and his path into Higher Education and teacher training would have been via vocational routes. Anthony seems to be calling on his previous experience (as expert) in this particular community of practice and also on his developing expertise in the practice of 'school teacher' to emphasise his identity as a part of the 'real world' e.g. 'my experience, my situation' (I 31). This contrasts with his view of the community of practice of mathematicians, as exemplified by the researcher, who inhabits 'a hypothetical world' (130) and who has departed from the real world e.g. 'you've gone out of a real life situation' (I 31). He stresses his view that the researcher/mathematician is going nowhere e.g. 'You're going right full back in circles to functions, that's something hypothetical - that's floating about, not related to anything or solving anything. It's not come from any real life situation, it's just a function, it's not

related to anything else.' (131-35). However he does seem to express some sympathy and desire for a greater level of participation in the practice of being a mathematician when he says 'I think that's why I have difficulty in seeing where it's coming from.' (135) This also seems to reflect his peripheral participation in this particular community of practice.

It seems that our interest at the outset of this study, in language and meaning, has given us a picture of some of the ways in which our students are working on re-learning algebra. However it has also revealed much more - a complex set of phenomena and questions with which to revisit both our data analysis and also the ongoing development of our own practice.

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