IS THE GAP BETWEEN GCSE AND A-LEVEL BIGGER IN MATHEMATICS THAN FOR OTHER SUBJECTS?

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Abstract

An analysis of the grades obtained by the age-18 cohort who took A-level examinations in 1994, matched with their results obtained at GCSE two years earlier, supports the idea that the 'gap' between GCSE and A-level is larger for mathematics thanfor English, except at the highest levels of attainment. Approximately one-third of students who get a grade A in a subject at GCSE and who go on to A-level get a grade A, whatever the subject. However, to have at least a 50% chance of getting a grade 'D' or better at A-level, one needs a grade 'B' at GCSE in mathematics, but only a grade 'C' *in English. As afirst approximation, the gap between GCSE and A-level is the same size in mathematics and Englishfor candidates who attain A-level grade A; one-half of a grade bigger in mathematics than in Englishfor candidates who attain A-level grades Band* C; *and one grade bigger in mathematics than in Englishfor candidates who attain A-level grades D and E.*

Introduction

This paper reports some of the results of a brief study (commissioned by the School Curriculum and Assessment Authority, and undertaken on behalf of the Joint Mathematical Council of the United Kingdom) that attempted to shed light on the size of the 'gap' between GCSE and A-level in mathematics. As well as the work reported here, the study had two other strands which looked at expectations of teachers of A-level and at the performance of candidates in GCSE and A-level examinations in different topics (Wiliam, 1996). The strand reported here compares the 'trajectories' of students who took GCSE in 1992 at the age of 16 and A-level in 1994 at the age of 18.

The transition from GCSE mathematics to A-level mathematics

Table 1 shows the results of the candidates in the age 18 cohort who sat an examination in single A-level mathematics in 1994. Of 22693 candidates who took A-level in 1994 after having got a grade A in GCSE mathematics in 1992, just over a quarter (26%) got a grade A at A-level, just under a quarter (23%) got a grade Band 18% were awarded grade C. Grades D and E were obtained by 13% and 8% respectively of those who got an A grade at GCSE. In total, therefore, 89% of those taking single A-level mathematics after getting a grade A at GCSE gained some kind of a pass in mathematics at A-level, although only two thirds (67%) got a 'good grade' (ie C or higher).

Candidates doing single A-level mathematics after getting only a 'B' in mathematics at GCSE fared much less well. Less than a third (32%) got a good grade and only just over two-thirds (68%) got any kind of pass at all. For those candidates attempting A-level mathematics after achieving only a grade 'C' at GCSE, the prognosis was even worse: one-sixth (17%) got a good grade and only one-half (51 %) got any grade at all. When one considers that many students with only a 'C' in mathematics are dissuaded or prevented from studying A-level mathematics at all (many schools and sixth-form colleges require that students have at least a grade 'B' or a grade 'C' on the higher tier at GCSE before being allowed to study mathematics at A-level), then grade 'C' in mathematics at GCSE can be seen to be a very poor predictor of success at A-level mathematics.

This approach excludes students who entered for double mathematics A-level but unfortunately the readily available data for double A-level mathematics gives only a single combined score (ie 10 for AA, 9 for AB, 8 for AC or BB, etc). One way of incorporating these data is to assume that the grades obtained by students entered for double mathematics are assumed to differ at most by one grade and that it is the highest grade that is the most representative. The result of combining these data with those for single A-level is shown in table 2.

		GCSE grades			
		A	B	C	D-G
A-level grades	A	26	4	2	7
	B	23	11	5	13
	C	18	17	10	19
	D	13	19	15	18
	E	8	17	18	12
	X	11	32	49	31
	Total	100	100	100	100
	Numbers	22693	12009	3310	314

Table 1: cross classification (%) of grades for single A-level mathematics and GCSE mathematics

		GCSE grades			
		A	B	C	D-G
A-level grades	A	32	5	2	9
	B	23	11	5	13
	C	17	17	10	19
	D	12	19	15	18
	E	7	17	18	12
	X	10	31	49	29
	Total	100	100	100	100
	Numbers	26461	12224	3334	326

Table 2: cross-classification of grades (%) for all A-level mathematics and GCSE mathematics

Even though the addition of the double-mathematics candidates changes very little, which of tables 1 and 2 is the most representative is a difficult question. Candidates doing double mathematics will typically have had twice as much teaching, which should increase their grade. On the other hand, the double mathematics students *are* part of the cohort, and, often being the most mathematically gifted students, would perhaps get an equally good grade had they studied only single mathematics A-level. The balance of the argument would appear to favour the inclusion of the double mathematics candidates, and so for the remainder of this paper, table 2 is taken as the most representative for the transition from GCSE mathematics to A-level mathematics.

Comparisons with other subjects

The foregoing analysis has shown that candidates with GCSE mathematics grades Band C do not fare well at A-level. This could be taken as evidence of a 'gap', but the results are difficult to interpret in isolation. For this reason, similar analyses were conducted for the transition from GCSE to A-level in English, and also with results aggregated across all A-level subjects.

The transition/rom GCSE to A-level in English

The situation in English is complicated somewhat by the existence of two separate GCSEsEnglish Language and English Literature-and by the existence of two separate A-levels with the same titles. This yields a total of four possible transition matrices. However, the number of candidates taking English language at A-level is rather small and the transition matrix for GCSE language to A-level literature is very similar to that for GCSE literature to A-level literature, and so the latter has been take as the basis for comparison with mathematics, and is shown as table 3.

As in mathematics, a grade A in English at GCSE confers a very high probability of success in the subject at A-level, although in English, possession of a grade B at GCSE does appear to confer a greater advantage over a grade C than is the case in mathematics.

		GCSE grades			
		A	B	C	D-G
A-level grades	A	32	6	1	1
	B	30	17	6	3
	C	21	27	16	11
	D	11	27	29	23
	E	4	16	28	30
	X	2	8	21	32
	Total	100	100	100	100
	Numbers	19411	18081	8062	600

Table 3: cross-classification of grades for A-level English literature and GCSE English literature

The transition/rom GCSE to A-level

The difficulty with a comparison of two subjects is that we don't know which is 'out of step'. Within the time-scale available there was not enough time to obtain and evaluate transition matrices for a sufficient range of other subjects, to provide a 'base-line' for comparison. However, the DFE's Statistical Bulletin 4/95 does cross-tabulate the average GCSE points scores obtained by students in the age-16 cohort in 1992 with the A-level points scores of the same candidates in 1994. Although in a far from ideal form, with suitable assumptions, these data can be used to provide a 'base-line' showing how GCSE grades are converted into A-level grades. The detailed calculations are given in Wiliam (1996), but are summarised below.

The first stage was to equate GCSE points scores with average GCSE grades. In order to do this, some assumption about the average number of GCSEs taken by each student needs to be made. Given that this cohort was subject to the requirement to teach all the national curriculum subjects for a 'reasonable time' in key stage 4, and given that this study excludes all students who did not sit at least one A-level within two years of taking GCSEs, it seems reasonable to assume that each student took an average of 10 GCSEs (see below for a discussion of the robustness of this assumption). Points scores were equated with grades by grouping all points scores with the nearest 'flat' profiles (eg 10 x B).

The second stage was then to equate the A-level points score with A-level grades. This was complicated by the fact that not all candidates took the same number of A-levels. In general, higher-achieving students took more A-levels. For each GCSE grade, A-level points scores for

'typical' candidates at grades E, D, C, B and A were generated by multiplying the number of A-levels taken by candidates with that GCSE grade by 1,2,3,4 and 5 respectively. Points-score boundaries were situated mid-way between the 'typical' scores.

The number of candidates in the points-score ranges were then allocated to grades in proportion to the amount of the points-range falling within that grade's range, resulting in table 4. In order to make comparisons easier, the data from tables 2,3 and 4 are shown as a cumulative frequency graph in figure 1.

		GCSE grades			
		A	B	C	D - G
A - level grades	A	32	21	5	3
	B	32	26	13	5
	C	22	23	18	13
	D	11	19	31	23
	E	2	7	21	31
	X	1	3	12	24
	Total	100	100	100	100
	Numbers	11818	50913	61714	33601

Table 4: cross-classification of candidates' grades for all A-levels and all GCSEs

Interpreting the data in table 4 is difficult, because of the number and the strength of the assumptions made, but the simplest interpretation is probably to conceive of it as showing how, on average, GCSE grades get converted into A-level grades. Of course, since student performance at GCSE is highly variable, and as students tend to select their better subjects for A-level study, table 4 does not represent how GCSE grades would be converted into A-level grades for all students for all subjects. Furthermore, the data in the table is highly sensitive to the assumption about the number of GCSE subjects taken. For example, if the average number of GCSEs taken by A-level candidates had been taken as 9, rather than 10, then table 4 would have been much more like the results for English Literature (table 3). Clarification on this point is necessary before any weight is placed on the data in table 4 and figure 1. However, since students doing A-level mathematics or English are unlikely to have chosen these subjects unless they were among their best subjects, table 4 probably does represent a reasonable basis for comparison with tables 2 and 3.

Comparative analysis

Each of the cumulative frequency polygons (henceforth called 'curves' for convenience) in figure 1 is characterised by two features. One is the 'level' of the curve: the curve for the grades obtained by candidates with grade A GCSE English is above that for the grades obtained by candidates with grade B GCSE English (which is of course what we would expect). The other feature is the inclination of each of the curves-the three curves for mathematics seem to be somewhat 'flatter' than the other six curves.

The simplest approach is to characterise the 'level' of the curve by how far along the horizontal axis the curve crosses the median (ie the 50th percentile), and its inclination by the gradient at this point. If we regard the five A-level grades as being underlain by a continuous scale (with A=5, B=4 and so on), we can derive quantitative estimates of the 'level' of each curve, together with a

measure of the spread of the A-level grades achieved (standard deviation). These data for the nine distributions are summarised in table 5.

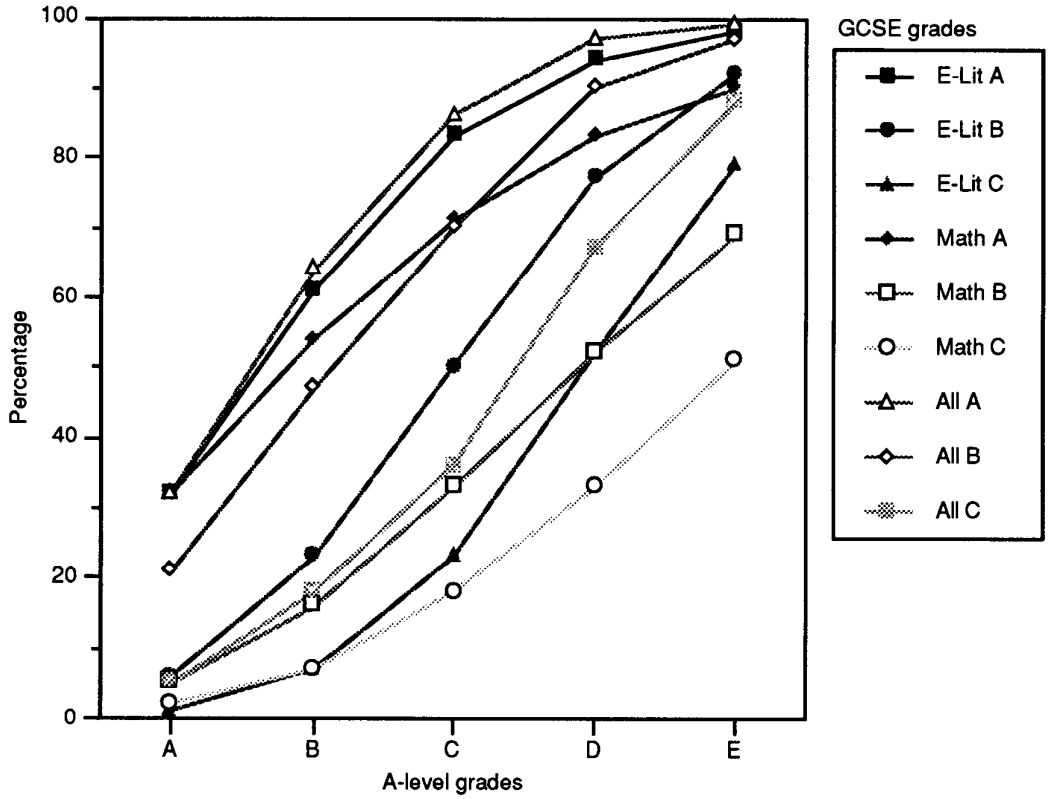

Figure 1: cumulative frequencies of average A-level grades by average GCSE grades

Figure 1 and table 5 between them summarise all the data generated for this study. There are many potentially interesting features, although it would be unwise to place too much reliance on these data, since they are based on only a single cohort's data, and very strong assumptions were needed to generate the aggregate data for the overall GCSE-A-Ievel transition. Nevertheless, several features are apparent from the data.

	Mathematics			English			Overall		
	A	B	C	A	B	C	A	B	C
level (grade at median)	4.2	2.1	1.1	4.4	3.0	2.1	4.4	3.9	2.5
slope (% points per grade)	22	19	18	29	27	29	32	23	31
mean A-level grade	3.31	1.74	1.12	3.68	2.47	1.62	3.79	3.26	2.14
sd of A-level grades	2.66	2.38	1.88	1.56	1.71	1.46	1.25	1.76	1.79

Table 5: summary data for the cumulative frequency curves.

Approximately one third of A-level candidates who had a GCSE grade A get an A-level grade A, whatever the subject. For the ablest candidates, therefore, it would appear that there is little evidence that the GCSE-A-Ievel gap is any bigger in mathematics than in other subjects.

However, at the lower A-level grades, the picture is rather different. For example, to have at least a 50% chance of getting a grade 'D' or better at A-level, one needs a grade 'B' at GCSE in

mathematics, but only a grade 'C' in English. In a very real sense, therefore, the gap between GCSE and A-level is one grade bigger in mathematics than in English for candidates who attain Alevel grades D and E. Interpolating between the graphs in figure 1 suggests that for candidates who attain A-level grades Band C, the gap between GCSE and A-level is one-half of a grade bigger in mathematics than in English. When the comparison is between mathematics and *all* A-levels, the gap appears to be the same size at grade A, one grade bigger for mathematics at A-level grades B and C, and one-and-a-half grades bigger at A-level grades D and E. However, due to the uncertain status of the data presented in table 4, this particular conclusion should be interpreted with caution.

Another important feature of the data is that, for English and mathematics, the curves for GCSE grade B are much nearer to the curves for grade C than to those for grade A. In both mathematics and English, the one grade difference between a grade C and a grade B at GCSE equates to a single grade difference at A-level (although this is from grade E to grade D in mathematics and from grade D to grade C in English, for the reasons discussed in the paragraph above). However, increasing a GCSE score from a B to an A improves the median English A-level grade by a grade and a half, and the median mathematics grade by *two* grades. A-level can therefore be thought of as placing a 'magnifying glass' on the highest grades at GCSE (A and B), but not on the lower grades.

A third feature of the curves in figure 1 is that the slopes of the curves for English are all very similar, as are those for mathematics, but the slopes for English are much greater than those for mathematics. This is related to the fact that the spread of A-level grades awarded to those obtaining a particular GCSE grade is much greater in the case of mathematics than English or of other subjects generally. In crude terms, GCSE predicts A-level performance better in English than in mathematics.

Summary

Because of the limitations of the methods used, firm conclusions are not possible. Nevertheless, the data analysed here do suggest that, for all but the very highest attaining students, the gap between GCSE and A-level is bigger in mathematics than in other subjects, which is perhaps surprising when it is borne in mind that current standards at GCSE are set so that substantially fewer candidates achieve good GCSE grades in mathematics than in almost any other subject.

References

Department for Education Analytical Services Branch. *Statistical Bulletin* 4/95. London: Department for Education.

Wiliam, D. (Ed). *The step between GCSE and A -level in mathematics.* London: School Curriculum and Assessment Authority (March 1996).