

IS THIS A SIGN OF THE TIMES? A SEMIOTIC APPROACH TO MEANING-MAKING IN  
MATHEMATICS EDUCATION.

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*This paper is intended to promote discussion around semiotics as a context for discourse in empirical and theoretical aspects of mathematics education. Semiotics is the study of the nature and action of signs and sign functions; the mathematics register consists of complex signs and should therefore be open to semiotic analysis. This paper introduces the basic elements of semiotics and discusses, in general terms, the possible implications of the application of a semiotic perspective to the context of mathematics teaching and learning. A number of examples from student's work in solving linear equations are included in order to illustrate some of the ideas put forward in this paper and to promote further discussion.*

There is an inextricable connection between signs and mathematics. One might even say that mathematics consist entirely of a complex system of signs, a position that is defensible from within a semiotic perspective. Semiotics is a point of view of the world that puts the sign at the centre of all human action, it is a doctrine of communication and intrinsic to the semiotic perspective is a dependence on the socio-cultural view of knowledge.

For some time now the mathematics education community has recognized the importance of the socio-cultural context of mathematics learning and such influences have become the object of study (for example see Eccles and Jacobs 1986, Walkerdine 1990 and Evans and Tsatsaroni 1994). Communication is the essential mediator of these socio-cultural influences and language is one (but not the only) sign system which effects this mediation. The role of language, both written and spoken, in mathematics has been investigated by Halliday (1978), Klemme (1981) and Pimm (1987) amongst others, and it is clearly evident from this work that language is instrumental in structuring and developing mathematical thoughts and actions. Nunes (1992) suggests further that specific cultural sign systems structure the organisation of mathematical activities but do not necessarily alter psychological functioning.

The increasing interest in cultural psychology is highlighting the work of Vygotsky (1977) for whom semiotics was intrinsic to his description of the development of higher mental functioning. Vygotsky suggested first that the word and later that tools were the mediators from the intersubjective to the intrasubjective and that concept development began in the social and was then internalised through the action of semiosis. Much of the work cited here as being of a semiotic nature takes the Vygotskian position on semiosis, or developments from that position. Semiotics did not begin with Vygotsky however, it has two roots; the first is in structuralism with the signifier-signified relationship of Saussure, the other is in pragmatism with the triadic action of signs developed by Charles Peirce. Peircian semiotics is more appropriate for a cultural interpretation as it is diachronic in nature, furthermore it is an appropriate starting place for mathematics educators as it has many commonalities with the Vygotskian approach. (For example Maffiolo (1992) draws parallels between the two approaches).

The question alluded to in the title of this presentation "Is this a sign of the times?" has two answers. The first pertains to the current position of semiotics in mathematics education, the second

to a specific example which will be discussed later. As the first answer to this question I would like to suggest that it may be possible for the socio-cultural to give way to the semiotic in the description of meaning-making in mathematics and that yes, in the current climate much work in the area of mathematics education is tending towards a semiotic perspective. In the remainder of this presentation I will outline some of the basic elements of semiotics and indicate the way in which a semiotics of mathematics learning may not only incorporate much of that which is deemed important but may add focus to research and to theoretical perspectives.

**Peircian Semiotics: The essential elements.**

Peircian semiotics differs from that of Saussure in that Peirce sees the action between signifier and signified or (to use his terminology) sign and object, as triadic. The sign stands for the object in relation to a third element, the interpretant. It is the interpretant which introduces the cultural and historical dimensions to semiotics, but it is not at all easy to understand its role. Vygotsky too considered semiotic action to be triadic yet for him the third element was the mediation means, the carrier of the meaning from external to internal. For Peirce the third element, the interpretant, although it may perform a mediating role is more than that. Peirce suggests that "[the interpretant] is all that is explicit in the sign itself apart from its content and circumstances of utterance" (Peirce 1906, quoted in Deely (1990) page 26.), and furthermore that the interpretant is a sign.

An example may serve to clarify the situation. Seigel and Cary (1989) describe a situation in which they show to a preschooler a box of toothpaste and ask the child what the writing on the box says. The child replies "brush teeth". In this situation the sign ( the box) stands for the object (toothpaste) in relation to the interpretant (the experience of having frequently brushed ones teeth after having seen the box), this is demonstrated diagrammatic ally in figure 1. Many other replies were possible, for example the child could have associated toothpaste with going to the dentist or going to bed, the sign could have indicated a number of different meanings to the child by providing for further context dependent interpretants.

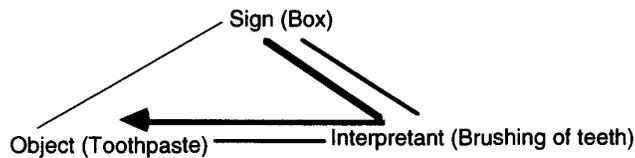


Fig 1

Interpretation of the sign may only take place by way of the interpretant, any direct connection between the sign and the object that is not recognised will remain devoid of 'sense'. The interpretant carries with it the socio-historical positioning of the interpreter and performs the functions of mediation and meaning-making, it is the single most important aspect of semiosis, for without it semiotic acts would have no meaning.

The introduction of the interpretant into the meaning-making equation also brings into question the nature of experience. Deely (1990) suggests that the semiotic universe divides the content of experience into three areas. The physical, the objective and the subjective. The objective is that which is experienced, the physical is that which is existent regardless of its being experienced and the

subjective exists only in relation to the action of semiosis in which something that is objective is recognised and becomes a sign. The physical environment is thus experienced by the subject through a network of objective relationships which are then reconstituted by a subject in a subjective domain into an objectively shareable world. Through the interpretant the action of semiosis defines a three dimensional meaning-space which transcends subject-object dualism. Semiosis takes place primarily in a social space and interpretants may be thought of as *cultural units* (Eco 1979) which both enable and are a product of a shared cultural understanding.

### **Vygotskian Semiotics**

Vygotsky (1978) saw the development of higher mental functioning resulting from progressive control over semiotic systems of increasing demand, in this description thought is "nothing more than the self-appropriation of the cultural space" (Maffiolo 1991). Vygotsky considered the word to be the cultural unit and saw the role of the word as central to the cultural line development and specifically to concept development.

Work of Becker and Varelas(1989) has investigated the Vygotskian notion of semiotically mediated development. They have conducted a study of counting activities with increasing degrees of semiotic demand noting that as development occurs the sign-object connection gives way to a sign-sign connection, with signs becoming more opaque as development progresses.

In his description of the development of scientific concepts Vygotsky (1977) invokes a semiotic process by which "the relationship to an object is mediated from the start by some other concept. The very nature of a scientific concept implies its position in relation to other concepts" (Vygotsky 1977, p.93). Scientific concepts are concepts that may be learned in a school context, things such as number and family relationships, and their nature is such that they depend upon other concepts (which Peirce would think of as signs) for their meaning. This is consistent with the Peircian notion of unlimited semiosis (Eco 1979) in which interpretants become signs which in turn become elements of further semiotic acts.

I would like to summarize this section with a lengthy quote from Maffiolo (1991) who sums up the semiotic position presented by both Peirce and Vygotsky in the following way:

With Peirce and Vygotsky, [we] recognise the subject as a sign and the sign as a cultural object, we place ourselves in a dialogical perspective, where thought is elaborated by and in the activity of communication, where cognition is constructed by and in the share of speech codes and contexts. This is in fact the opposite of traditional epistemology, which assumes that human beings use signs to communicate the thoughts they previously and individually constructed according to natural and/or universal processes. [sic] (Maffiolo 1991 p. 491).

### **Semiotics and Mathematics Education**

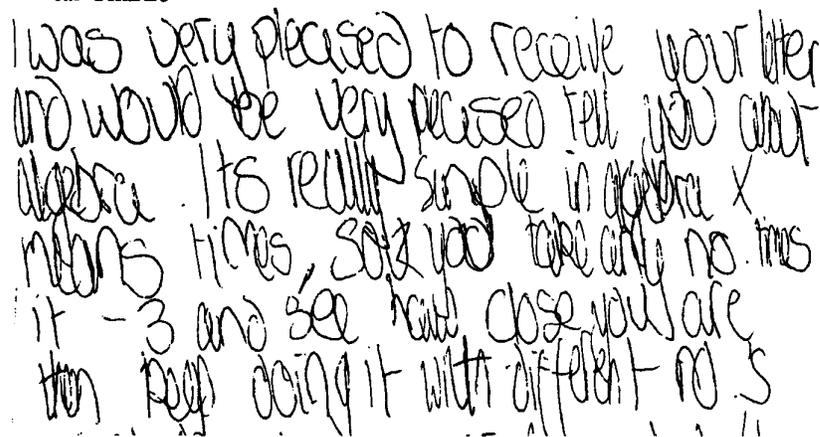
What can semiotics offer mathematics education? would be the question in most need of an answer. I would like to suggest that semiotics may be of use in both a reconceptualisation of theoretical perspectives and in terms of empirical methodologies. Theoretically, semiotics may offer a description of the meaning-making process that accounts for the socio-cultural nature of experience and, avoiding Cartesian dualism and with it problems with intersubjectivity (Lerman 1996), provides an alternative description of the social/personal interface. Empirically, focus on the role of the sign,

and in particular the nature of the interpretant, may assist in the understanding of the meaning-making process in specific contexts. Following the work of Becker and Varelas (1989) the nature of semiotic development may give some insights into the ability of students to carry out tasks with varying semiotic demand. This work would necessarily build on work carried out in specific semiotic systems such as that of Klemme (1981), Pimm (1987), Rotman (1988), Stage (1991) and Nunes (1992), but may serve to unite such work under a comprehensive framework.

I would like to offer for discussion some examples from my own investigation into the possibility of a semiotic description of meaning-making in a specific context. The following extracts come from students' writings taken as data from part of a three week study into the development of meaning in the learning of linear equations. It is at this point I will leave it up to you to answer the question, is this the sign of the times?

The following responses are to the question: What does  $x$  mean in the equation  $x-3=5$ ?

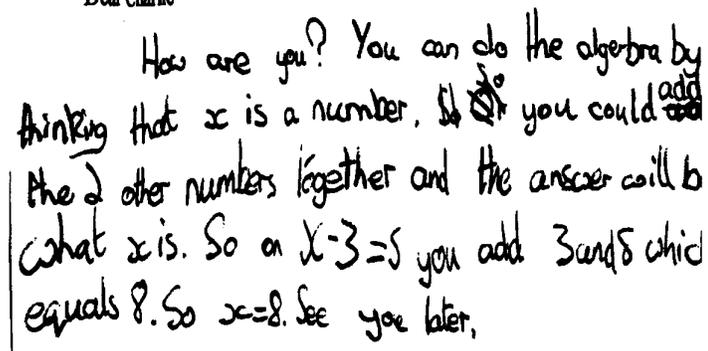
Dear Charlie



I was very pleased to receive your letter and would be very pleased tell you about algebra. Its really simple in algebra  $x$  means times, so you take away no. times it  $-3$  and see how close you are then keep doing it with different no.s

A semiotic analysis of this and the following response would be concerned with the interpretants of the sign  $x$ . In the first extract the sign  $\langle x \rangle$  is seen and recognised not as its intended meaning (as a general number or an unknown) but as the sign for "multiply". Why is that? One might conjecture that the equation is putting too much semiotic demand upon the student and that she is unable to make meaning, except by considering the sign  $\langle x \rangle$  in terms of a previous network of experiences involving multiplication. One may also say that as she has not recognised the sign  $\langle x \rangle$  she has not been able to make a meaning for it and thus resorts to a trial and error method (of which it seems that she has had experience).

Dear Charlie



How are you? You can do the algebra by thinking that  $x$  is a number, so you could add the 2 other numbers together and the answer will be what  $x$  is. So on  $x-3=5$  you add 3 and 5 which equals 8. So  $x=8$ . See you later,

The second extract shows that the student sees  $\langle x \rangle$  in terms of a single number, relating it to previous experiences of sums like  $\langle 8-3=5 \rangle$ . But more, this student also seems to recognise a concept, (or is it a process?) in which he may manipulate the equation, having made some meaning for the sign  $\langle = \rangle$ , and extract the number.

Consider the next two extracts in which the semiotic demand of the equations is far greater than those of the previous equation.

I had a bit of trouble at first but now I understand.

$$3y+2 = 7+6$$

$$3y-4 = 6-2$$

$$2y = 4$$

$$y = 2$$

The reason why I have put the y's on one side is because I can do that but if it's  $-y$  and I wanted to change sides the  $-$  would become  $+$ . You are trying to look for  $y$  not  $3y$  but  $y$ . It doesn't ..

The above extract shows a student manipulating, almost by rote, an equation. The student writes "I had trouble at first but now I understand" but what does the student understand? One might conjecture that his network of experiences is such that the interpretant that he has for the sign, which is the equation, manifests itself by way of a process. The last extract demonstrates a student using a trial and error method and one could ask what does the equation mean to that student? Certainly this demonstrates a case of a student trying to solve an equation with a high level of semiotic demand yet using a technique with a much lower level of semiotic understanding.

Q4. Solve for A  $7+2A=4A-11$

~~1 2 3 4 5 6 7 8 9 10 11 12 13 14 15~~

Answer A=9

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Q5. Find Q in the equation  $3(4Q-11)=5(2Q-4)$

$$3 \times 4 \times \underline{\quad} - 11 = 5 \times 2 \times \underline{\quad} - 4$$

~~1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25~~

Answer Q=5

I hope that these few examples show that, as a framework, semiotics has potential for the description of the meaning-making process. Further I hope that the discussion of the theoretical aspects of a semiotic point of view will be sufficient to generate interest from the mathematics education community in the possible application of a semiotic framework to the description of meaning-making in mathematics. Evidence from the study alluded to in this work suggests that the notion of semiotic development is an important one in that students who have been able to act on equations with a high level of semiotic demand successfully are those who have been able to build a meaning for the signs in that context that is of a high level of sign-sign foregrounding. For example consider the two extracts above, in the first extract the student can solve equations of the form

$\langle a+x=b+cx \rangle$  whereas the student in the second example is using an inappropriate method, and a method at a much lower level of semiotic development. A further implication is that development would be facilitated by the identification of meanings useful for progression and by encouraging those students to make those meanings at a lower level of semiotic demand than at which they may be required.

I would like to conclude by noting that in the current climate mathematics educators are moving towards a description of mathematical meaning-making that involves society, culture communication and context, and that many are moving further to investigate the role of the sign in mathematical meaning-making. Semiotics, at least in the version presented by Peirce and Vygotsky, exists as a well developed point of view of the world that embraces all of these elements in a way that transcends traditional philosophical objections and as such may prove (and indeed is already proving) to be a fruitful perspective for the description of meaning-making in, in particular, mathematics education.

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