Constructing Meaning for Number

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This paper gives a brief account of a research study in Boxer, in which primary school children (aged 10 - 11) first constructed and then interacted with an operational number line. The episodes chosen here illustrate how a discourse framed round this computational object enabled them to construct meaning for number.

Introduction

There is now an accumulated body of research both here and abroad (Brown, 1981; Carpenter et aI., 1981; Foxman et aI, 1985) which demonstrates that children lack meaning for decimals. There is also a recognition of the difficulties which children encounter with directed numbers (Bell, Costello and Kiichemann, 1983). The challenge for this study was to investigate how programming their own number line would mediate primary school children's understandings of both decimals and directed numbers. In what follows, the programming, or construction, stage is omitted. Each of the episodes illustrates children's interactions with the fully operational number line[1]•

Episode 1: Seeine Directed numbers as a process

This episode has been chosen to show how their Boxer number line mediated Jacky and Kirsty's expressions of directed numbers.

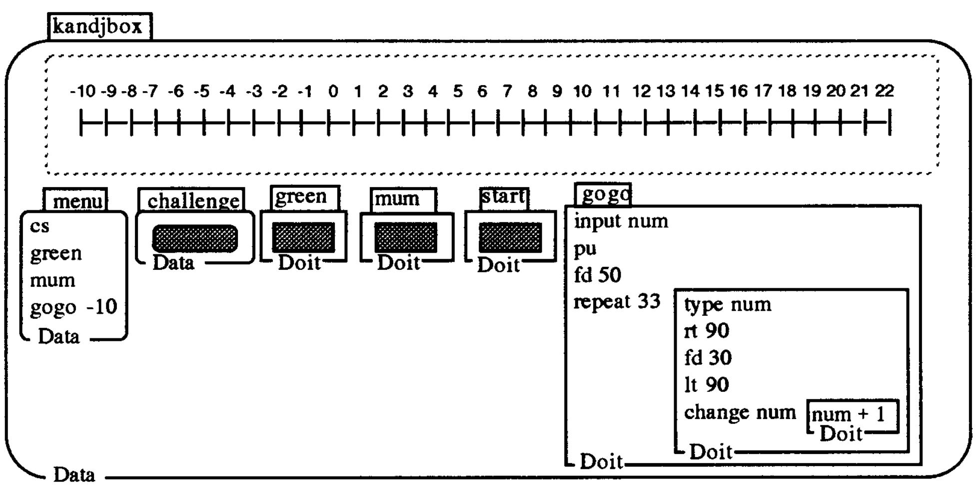

Figure 1: Visualising Directed Numbers with Kirsty and Jacky's Number Line

Extensions to negative numbers was often provoked by interventions which built on the students' successes with natural numbers. With the screen number line displaying the numbers 0 to 32 in steps of 1, I asked Jacky and Kirsty what would happen if they started at -10 instead.

1 For the benefit of readers who might not have seen Boxer before, the doit boxes are the programs. In Figure 1, for example, 'mum' is Jacky and Kirsty's program to draw the number line, while 'gogo' sets the starting number and then types the numbers above the line.

Extract 1:

Speaker	Dialogue	Commentary
Interviewer:	What do think will happen if we start at minus ten and go up in ones?	Figure 1.
Jacky:	Don't know.	
Interviewer:	No ideas? [no response] Okay, just put minus ten.	girls run 'mum' and 'gogo -10'
Interviewer:	Minus ten, minus nine, minus eight, minus seven, minus six ...	Reads from left to right on the number line.
Jacky:	There's a nought there as well.	Points to screen number line.
Interviewer:	There's a nought, one, two, and all the rest.	The number-line was calibrated in steps of 1 from -10 to 22 (Fig 1).
Jacky:	If we do minus half of thirty two. That's sixteen. If we do minus sixteen, then it will be sixteen each end.	Points to both ends of the number line in the graphics box.
Interviewer:	Try it.	

It is hard to imagine this fonn of reasoning occurring in a traditional setting. I would suggest that it might not occur as readily with conventional software either. It would seem that Jacky's insight into how this computational object affected the behaviour of numbers was influenced by the fact that she had constructed it in the fIrst place. It is worth noting that these two children were chosen to represent the bottom quartile of the year 6 ability range.

Episode 2: From fractional to decimal expressions

This episode has been chosen to show how Leroy and James (who represented the middle of the year 6 ability range) used a notation which they were familiar with (fractions) to express a quantity that they lacked familiarity with (a number between 0.1 and 0.2).

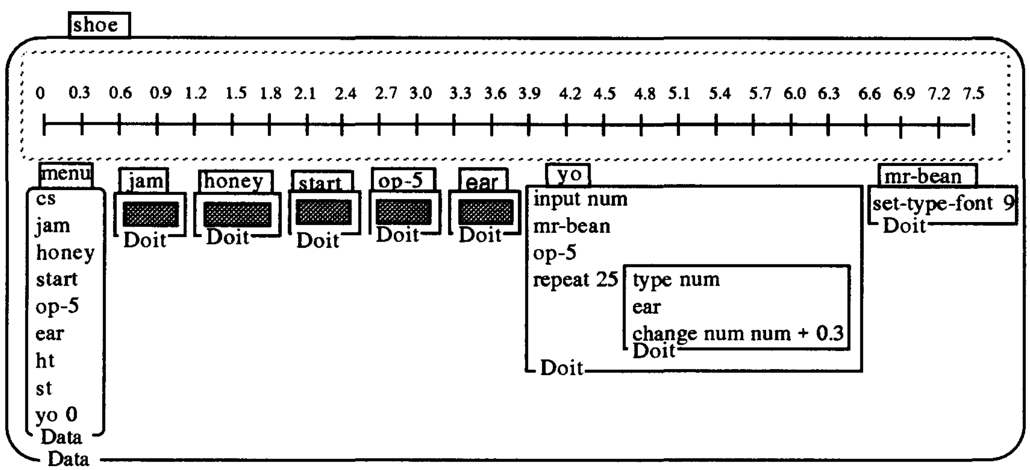

Figure 2: Visualising the Number Continuum through Leroy and James's Number Line

Extract 2:

Speaker	Dialogue	Commentary
Interviewer:	Suppose you started at nought and you wanted to finish at about three point five.	The number line was graduated from 0 to 7.5 in steps of 0.3.
Leroy:	Where's that?	Points to the on-screen number line.

Interviewer:	Roughly in the middle there.	Points to the centre of the on-screen number line.
Leroy:	Try point one.	They run this.
Interviewer:	It looks like point one is a little on the small side.	After getting a number line graduated from 0 to 2.5 in steps of 0.1.
James:	Point one and a half.	
Interviewer:	How would you write down what James just said: point one and a half?	Hands Leroy a piece of paper. Leroy writes: $.1^1/_2$.
Interviewer:	Would you mind changing that point one to point two?	Points to increment. The boys change this and run the program.
Interviewer:	Point one wasn't quite right and point two wasn't quite right .	Refers to new screen number line ending at 5.0
James:	In between point one and point two.	Refers to the value of the increment in the program line.
Leroy:	Point one and a quarter.	
Interviewer:	Explains convention of point one and a half as point one five.	
Leroy:	Can we try it?	They run it with 0.15 and then with 0.14 which gives the exact answer 3.5.

Rather than encountering difficulties with 'in-betweeness' as res~ch has shown is frequently the case with conventional media (Hart, 1981, Markovits and Sowder, 1991), the Boxer number line seemed to pre-dispose students to the existence of such numbers.

Episode 3: Accuracy of Expression

This episode has been selected to show how Julia and Neil's Boxer number line motivated them to persist in seeking a solution to a challenge (equivalent to solving the equation 24x = 1) to the extent of going to 4 decimal places. It suggests that the accuracy of the expression in this case serves as a measure of their understanding.

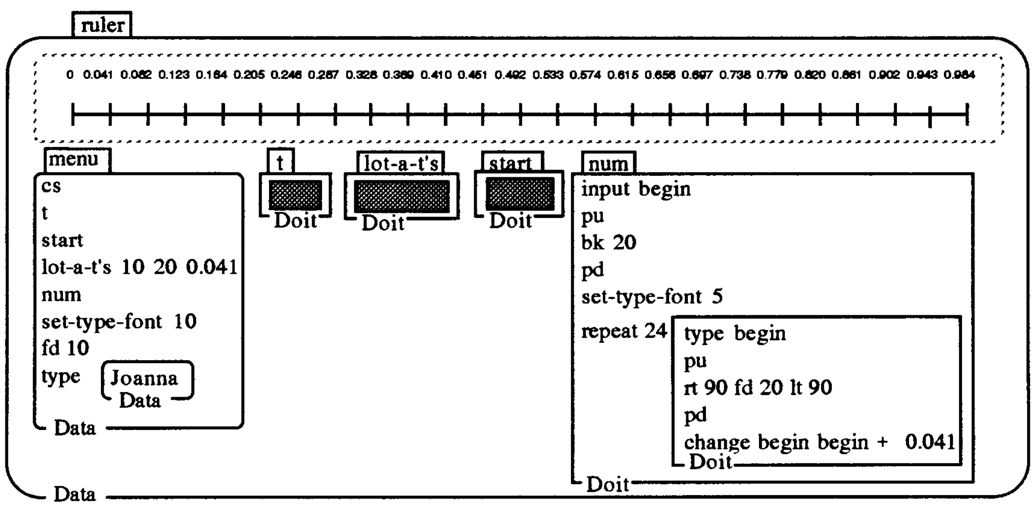

Figure 3: Expressing small decimal quantities with Julia and Neil's Number Line

Extract 3:

Speaker	Dialogue	Commentary
Interviewer:	Finish at about one.	Note the approximate nature of this and later questions.

Julia:	Try point zero four. Maybe point nine six.	Before running program.
Julia:	One!	The program had rounded up. Boxer's printing-precision needed to be adjusted.
Julia:	Ah. Zero point nine six.	After the adjusted program had been re-run.
Interviewer:	Could you adjust that to end up even closer to one.	Refers to the increment 0.04 in the line: 'change begin [begin + 0.04]'
Neil:	Changes increment from 0.04 to 0.041.	Figure 3.
Both:	Point nine eight four.	Reads end number on new line. Both students pleased by answer.
Julia:	Point oh four two.	The number line ends at 1.008. They then go on to try .0418, .0417, .0416, then .04168. and .04167.
Both:	One point oh oh oh oh eight.	At each stage, the students appear to be very pleased with the degree of accuracy that they are obtaining.

Julia and Neil, who were representative of the top of the ability range, were able to program the number line to do what they wanted it to do. This gave them power over decimals, so they could directly manipulate and visualise numbers with three and more decimal places with an ease that would not have been possible in conventional media. It then became an intellectual challenge to seek greater degrees of accuracy than the task required.

Conclusions

These three episodes do not prove anything. Rather, they illustrate the kind of thinking that seemed to be engendered through working in the dynamic medium of Boxer. In particular, the constructive activity of programming their own computational objects gave children insight into how those objects worked, and this seems to have been crucial in their subsequent ability to connect changes in the program's parameters with the behaviour of decimals and directed numbers. The fact that these primary school children were able to construct such meaning warrants, I believe, studies of a similar nature.

References

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