LANGUAGE AND STRATEGIES IN CHILDREN'S SOLUTION OF DIVISION PROBLEMS

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*Year 5 and 6 children have been videotaped while solving a variety of written division problems. We have examined their strategies and found that children with more flexible strategies are often more successful at solving these division problems than children who use only one or two strategies to solve them.*

*Using examples on video, we shared some findings of this ongoing research aimed at examining the development of these strategies and the use of language over a period during which children meet division formally in school.*

Introduction

There are many ways which pupils might describe, interpret and solve a division problem in written, symbolic form, like '96+4'. They might, for example, describe it using the words 'shared by' or the word 'divided by'. Indeed different *words* might indicate different *meanings* or *interpretations* assigned to the problem. 'How many fours in 96' or '96 shared into fours' indicates that the pupil is interpreting the question as 'how many groups or lots of 4 are there in 96?' (measurement model). On the other hand, '96 shared by 4' or '96 shared between 4' indicates that the pupil is solving the problem 'if 96 are shared among 4, how many items would be in each group?' (sharing or partitive model). The 'sharing' interpretation of division, although only one possible interpretation, often persists after division has been formalised at school, possible because experience of both the language and the concepts of division is often grounded in sharing experiences even before formal schooling.

However, there are a variety of visual images, interpretations and outcomes that might be associated with division problems, and it is even possible that pupils will negotiate meaning amongst themselves. In the case where pupils are not given a context in which to interpret written, symbolic problems, there are also many different ways in which they could approach *solving* these problems. In dealing with the numbers pupils could, for example, begin with the divisor and count up to the dividend, or they could begin with the dividend and somehow work their way down to the divisor. They could also approach or represent the problem using other, broader strategies like drawing a picture, making tallies, or creating a story.

Aims and Expectations

One of the aims of our research project is to classify such strategies that pupils use when solving six particular problems, presented below in Table 1. No context was provided for these problems. We would expect that the pupils with greater flexibility and therefore a greater range of interpretations and strategies to choose from, would achieve greater success in these problems. This is because the problems have been designed to require different strategies, and also to challenge common 'primitive', intuitive beliefs about division that have been found to exert unconscious control over solution processes and thus affect performance (Fischbein, Deri, Nello & Merino, 1985). For example, the problem 4+1;2 challenges the belief that 'division makes smaller', or that the quotient is always smaller than the dividend, and also the belief that the divisor must be a whole number (a belief strongly associated with the partitive model of division). The problem 6+ 12 challenges the belief that the divisor must be smaller than the dividend. We are particularly interested in pupils' responses to and ways of dealing with these problems.

Sample and Methodology

Our sample consisted of 54 Year 5 and 6 children from three schools, a city primary school, a rural primary school and a middle school. They were selected to work in pairs as we hoped there might be some interesting communication or even negotiation between the pairs which might shed light on their interpretations and/or solution strategies. The pupils were video-recorded while solving the problems and then explaining how their solutions. While we acknowledge that verbalisation of strategies may disrupt or even change the solution process (Ashcraft, 1990) and is particularly difficult for some pupils, the strategies and meanings indicated by verbalisation are believed to be very valuable.

Results

Table 1 shows the success rate on the various problems. As expected the success rate on the problems which challenge intuitive beliefs about division (4+1;2 and 6+12), was relatively low. Perhaps more surprisingly, success with 34 + 7 was rare, mainly owing to a lack of understanding of what we, the members of the formal mathematics community, understand by 'remainder'; a common answer for this problem was '5 remainder 1 '.

| | Number successful | Number attempted | % Successful |
|---|---|---|---|
| 96÷4 | 28 | 54 | 52% |
| 34÷7 | 6 | 54 | 11% |
| 6000÷6 | 33 | 54 | 61% |
| 4÷½ | 20 | 54 | 35% |
| 6÷12 | 10 | 52 | 19% |
| 68÷17 | 14 | 48 | 29% |

**Table 1: Success on Each Problem**

Coding of the strategies used by the pupils was, to a certain extent, difficult as shown by our video evidence. There is inevitably some overlap, for example between some categories and the 'M' category which simply overlaps that we do not have access to the pupils' mental images and strategies. The following table shows our classification.

| Strategy | | Example | | |
|---|---|---|---|---|
| CU<br>Counting up | | Danny<br>(Year 6) | 34÷7 | I tried to do 7 into 34; the 7 times table up to 28...no more sevens would fit |
| PD<br>Partitioning the dividend | | Naomi<br>(Year 5) | 96÷4 | 80 divided by 4...the answer was 20. 16 divided by four, so the answer was 4. |
| A<br>Algorithmic approach | AW<br>written algorithm | Paul<br>(Year 6)<br><br>Matthew<br>(Year 5): | 96÷4<br><br>¹9'6<br>÷ 4<br>24 | I did four...um...I put a line like that...and I put 96 under, under there, and I put 4 over there, and I said how many fours are there in 9, and that's 2 remainder 1. So I put the 1 just above the 6, and I made that 60, and I said how many fours in the 16, and it's 4. |
| | AD<br>dealing with digits | Daisy<br>(Year 5) | 6000÷6 | There's one 6 in 6, no 6's in nought, so you just put three 0's at the top. |
| | LE<br>logical error | Melanie<br>(Year 5) | 68÷17 | 10 divided by 60 is 10. Put the 8 divided by 7 is 1...one remainder. 11 remainder 1. |
| AF | Associated number fact | Melanie<br>(Year 5) | 96÷4 | 100 divided by 4, take away 6...I mean take away 4. |
| M<br>Mental strategy or image | | Naomi<br>(Year 5) | 6÷12 | I knew that 12 was double 6, so the only way to fit 12 into 6...would be 12 halves |
| O<br>Other | Oa<br>approximation | Daisy<br>(Year 5) | 68÷17 | Two seventeens are about thirty-something, so it's about 4. |
| | Ot<br>trial and improve | Elizabeth<br>(Year 5) | 45÷5 | I took a guess, I like said te...11 or 12, then added 12 five times, and the answer was...well, it wasn't right, so then I took...it was more than the answer, so I like had 11 and 10....then I wrote down 9 and it worked out the right answer. |
| | Or<br>rule | Johanna<br>(Year 5) | 5÷1 | It's hard to explain but you try to divide something by 1, you end up with the number you got to start with. |
| | Om<br>memory | Hayley<br>(Year 5) | 6÷12 | Well, I've heard the sum before...so I didn't exactly work it out, I just remembered, and I wasn't really sure about the answer. So...and I thought, when I looked back into one of my other lessons I thought they said the answer was 2. |
| | Op<br>pattern | Laurence<br>(Year 5) | 4÷½ | Well, because two... if it was divided by two they'd get 2, if it was divided by 1 they'd get four, if it was divided by half they got 6 or 8....if it went into 2 times table. |
| I/?<br>Indecipherable strategy, guess or strategy unclear | | Sammy<br>(Year 5) | 68÷17 | I divided 17 into quarters and that's 7 and 7 and 7 and 7. And 14...7 and 7 makes 14 and then another 7 and 7 makes 14. So you put the 14's together and that makes 28 and you add the 10 from the 17 on and that makes 38. |

| | | | | |
|---|---|---|---|---|
| MP Misreading problem | SN Switching the numbers | David (Year 5) | 6÷12 | (It's 2) because there's two 6's in 12's. |
| | DS Doing a different sum | Michael (Year 5) | 34÷7 | I crossed out the (3) and gave a 1 to the 4 so it's 14, then I had...take away 7, and I had 7 and then I just put the 2 from up here down here. |
| | E Error in reading problem | Neil (Year 5) | 4÷½ | 4 divided by one and a half, so one person has one then the other one has one, then there's 2 left, then you give that one to the other person and to the other so it's 2 and a half of one. |

Table 2: Coding of Strategies

Additional ways of representing, seeing or dealing with the problem, as opposed to dealing with the numbers, included drawing pictures, making tallies (which accompanied either the 'Counting Up' strategy or the 'Partitioning the Dividend' strategy) and creating a context or story, for example Ellie (Year 5), solving 6000+6: You've got 6 people, give them each 1000.

Table 3 shows the frequency of use of the various strategies in *successful solutions* of the six problems. More than one strategy is often indicated for the same solution attempt by the same child. For example, written methods would often accompany one of the other strategies.

| | CU | PD | A | AF | M | I/? | O | Total* |
|---|---|---|---|---|---|---|---|---|
| 96÷4 | 10 | 9 | 9 | 1 | | 1 | 1 | 28 |
| 34÷7 | 6 | | 3 | | | | | 6 |
| 6000÷6 | 1 | 4 | 13 | | 18 | | 3 | 33 |
| 4÷½ | 15 | | | | 7 | | | 20 |
| 6÷12 | | | | | 10 | | 2 | 10 |
| 68÷17 | 13 | 1 | | | | 1 | | 14 |
| Total | 45 | 17 | 25 | 1 | 35 | 2 | 6 | |

Table 3: Frequency of Strategy Use in Successful Solution (* Total Successful Attempts)

As expected, success required flexibility in strategy use. In the solution of 96+4, 32% of successful strategies involve an algorithmic approach, 32% of successful strategies involve partitioning the dividend and 36% of the successful strategies involve counting up in groups of the divisor. In the solution of 34+7, all of the successful strategies involve counting up in groups of the divisor. In the solution of 6000+6, 55% of successful strategies involve mental strategies or images and 39% of successful strategies involve standard written algorithms or 'dealing with digits'. In the solution of 4+'l2, 75% of successful strategies involve counting up in groups of the divisor. All of the successful strategies for 6+12 involve mental strategies or images. In the solution of 68+ 17, 93 % of successful strategies involve counting up in groups of the divisor.

Lucy (Year 5) attempted a strategy of counting up in groups of the divisor for all the problems except 6+12, although she did attempt one additional strategy (written algorithm) in the case of 68+ 17. She was not successful in any of the problems, and of course faced enormous problems when trying to solve 6000+6 by counting up in sixes! On the other hand, Rebecca (Year 6), was successful on all tasks except 68+ 17, on which she made an arithmetic error. She used a standard written algorithm for 96+4, counting up in groups of the divisor for 34+7, *4+Y2* and 68+ 17, and a mental strategy with creating a context for 6000+6. She initially switched the numbers of the sum 6+ 12, getting two as her answer, but then created a context and reached the correct answer. These examples suggest that pupils who are flexible in their solution strategies may achieve more success than those who attempt to use only one approach.

Another interesting phenomenon which we observed was the inability of some children to keep track of the (sometimes relatively sophisticated) strategies which they began. A good example of this is Lauren (Year 5) solving 96+4. "Well what I did was I, urn, urn, sorry .. .! did 4 divided by 10 first, which was, urn, no 10 .. .! know what I did. I did 4 divided by 40 which was 10 ... no, sorry, I meant 40 divided by 4 which was 10, and then, so I put 10 down, and then I did 4, 4 divided by 80, no 80 divided by 4 which was 20, and then, and then., then I added 16 on, and that's what I got. 46". She was attempting to partition the dividend but lost track firstly while splitting up 80, and then by simply adding 16 on at the end. This indicates a limited cognitive short term memory, overloaded while trying to solve number facts and the problem as a whole simultaneously.

There was evidence of some common misconceptions, for example Ellie (Year 5) saying "Y ou can't do 6 divided by 12". These sometimes resulted in the pupils doing a different sum, for example Ellie (Year 5) worked out that 4+1;2 was "two because that's a half of 4". This error was made in 38% of unsuccessful attempts at this problem. A common error (which occurred in 83% of unsuccessful attempts) for 6+12 was reversing the two numbers, for example Danny (Year 6):
"Can't do 6 divided by 12, so I did 12 divided by 6. I switched things around ... did 12 divided by 6". Table 4 shows the frequency of these reversal of the roles of numbers. In addition, 35% of unsuccessful strategies for 68+17 involved the logical error, usually in combination with a standard written algorithm.

| | LE | Misreading | | | Total* |
|---|---|---|---|---|---|
| | | SN | DS | E | |
| 96÷4 | | | 1 | | 26 |
| 34÷7 | | | 1 | | 48 |
| 6000÷6 | | | 2 | | 21 |
| 4÷½ | | 6 | 13 | 2 | 34 |
| 6÷12 | | 35 | 1 | 1 | 42 |
| 68÷17 | 12 | 2 | 2 | | 34 |
| Total | 12 | 43 | 20 | 3 | |

**Table 4: Frequency of Use of Common Incorrect Strategies and Errors in Unsuccessful Solution Attempts (*Total Unsuccessful Attempts)**

## Remarks and Conclusions

Perhaps the most remarkable finding is the diversity and richness of the pupils' strategies for solving our division problems, and the relative sophistication of some of these strategies (see also Murray, Human & Olivier, 1991; 1992). Our classification of strategies has also allowed us to investigate the success of various strategies for the different problems, and we have found that a fair amount of flexibility is required to solve the various problems, in line with Gray and Tall (1993) who state that pupils who use counting procedures which fail to generalise when greater sophistication is required, are actually doing a more difficult kind of mathematics and thus not succeeding.

It is also interesting that pupils respond to problems in which the numerical data challenges their intuitive beliefs by changing the roles of the numbers involved. Thus close adherence to limited interpretations of division can inhibit solution procedures. The need for challenging these beliefs as well as encouraging greater flexibility of interpretations and strategies may perhaps point to the need for discussion and negotiation in the classroom.

The data which we reported on here was simply related to strategies, while work on pupils' language in describing these problems is ongoing. Weare also doing a limited study of pupils' development by presenting the same pupils with similar problems after they have once again met formal division. Teacher questionnaires are being used to collect information on the language, representations and concepts which the teachers aim to introduce. We welcome any comments and suggestions.

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