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RESTRUCTURING THE DEVELOPMENTAL ALGEBRA CURRICULUM: A PROBLEM–CENTERED LAB APPROACH

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This article reports the results of a longitudinal study which compared the performance of college students taught introductory algebra using non-traditional materials to the performance of students taught using traditional materials and methods and their success in subsequent courses. The non-traditional materials focused on the concept of function, with skills taught in context and the use of graphing calculators integrated throughout the course.

In the United States, over 80% of mathematics course offerings at two-year colleges specifically address the needs of college students who plan to pursue careers that do not depend on knowledge of calculus or upper division mathematics, or of students who need calculus but enter college unprepared for mathematics considered "college-level." In the 1990/91 College Board of Mathematical Sciences Survey (CBMS), 56% of the 1,295,000 students studying mathematics in two–year college mathematics departments were enrolled in remedial (developmental) courses, i.e., Arithmetic, Algebra, Intermediate Algebra, Geometry (Albers, et al., 1992). For students who begin their college mathematics program in courses at the developmental level, a majority will not complete a college–level mathematics course. Historically, at Harper College, less than 15% of students who initially enrolled in an introductory algebra course completed a college level (transferable) course within three years of their enrollment in the developmental program.

A restructured introductory algebra pilot course was introduced at William Rainey Harper College in Spring, 1992, using non-traditional materials designed to support collaborative group work and active student participation, focused on the concept of function, with skills taught in context, and integrating the use of technology. The belief that a student learns mathematics by working in a social context to construct mathematical ideas and reflects on these constructions to make sense out of problem situations is a basis of these materials. The pilot materials cover most skills traditionally included in an introductory algebra course. The development of decision-making and problem-solving is emphasized. Students are encouraged to make good choices about the techniques that make the most sense in a given situation and to check answers for reasonableness and accuracy.

This project was awarded a National Science Foundation grant in April, 1994 to continue development of the curriculum materials, design and implement dissemination models, and to develop a network of field-testers. This past year, nearly 800 students in developmental studies programs at community colleges, four-year schools and universities have used the pilot introductory and/or intermediate algebra materials.

During the 1994/95 academic year, pre- and post-test instruments and attitude surveys were developed and tested. Three sites plus Harper College provided preliminary data about student attitudes and student characteristics. Over forty percent of the students who completed the surveys did not have a mathematics course in more than a year prior to enrolling in the pilot course. Only thirty percent of the students perceived of themselves as good algebra students, with almost twenty percent saying they were disastrous mathematics students. Nearly eighty-eight percent of the students surveyed had never used a calculator and sixty-three percent had never used a computer to do mathematics. The average number of hours students were enrolled in classes was 11.7 semester hours. Enrollments ranged from three semester hours to 20 semester hours per week for some students. Twenty-five percent of the students surveyed worked 26 or more hours each week on an outside job.

Student reactions to the reform materials have varied from extremely positive to antagonistic and resistant. In general, older returning students (over 22) are experiencing greater success than students recently graduated from high school (17-22). An older female student during a mid-term site visit last October insisted on being interviewed. She informed us that the pilot algebra materials allowed her to finally experience success in a mathematics class and she felt that she had "mathematical power." This student had attempted algebra three times previously and had dropped the course each time prior to completing the course. Her children had paid her tuition for the semester as they wanted her to try once more to realize her dream of becoming a nurse. During the interview, this student stated that for the first time in her life she knew she could do mathematics–and that it was impacting her success in other classes. Not only was she doing well in math class but she was getting an A in Chemistry as well. She stated that she was finally able to convert Fahrenheit into Celsius and she understood why the conversion worked. "Formulas no longer give me any problems because now they make sense."

Interviews and written self-evaluations at the end of the term indicate that many students feel that they have developed more self-confidence in their ability to do mathematics. A typical student reply is:

> I have learned to approach a problem in several ways. There's usually more than one way to solve a problem and I've learned to stop and think about that before I start to work the problem.

Collaborative group work has been well received by many students. The students in our classes are generally commuter students, with heavy work commitments outside of school. Still, many of these students find times that are compatible with the schedules of other class members and establish study groups outside of class. At one field site, a group of students regularly did their homework via a nightly conference call by telephone with all four group members on the line to share in the discussion and debate. At another site, the entire class met at a member's home on a lake to do a take-home exam one beautiful Saturday last October. This class brought along spouses and children. While the class members worked in groups on the exam, spouses and children went swimming and/or fishing. After finishing the exam, the class and their family members had a barbecue.

During a recent workshop held at one of the field sites, seven students who had taken either one or both pilot courses agreed to discuss their experiences. They were asked by workshop participants how they dealt with students in their groups (their peers) who did not contribute to the group effort. Student responses ranged from citing the efforts of class members who encouraged two members to stay in class and "work at it, it will come together," (both did) to the comments from all seven indicating that the groups monitored their own members and refused to work with those who frequently came unprepared. During the discussion, the students made a strong plea to teachers saying "Don't keep slowing down for those students who never do their work. We get bored. We do our work and that work should be honored by you. We want our money's worth. Teach those of us who come to class prepared and are willing to do the work."

In addition to the text materials, various alternative forms of assessment including journals, small-group take-home exams, and concept maps were used. The journals were designed to provide on-going feedback about student understanding and difficulties on a weekly basis during the term. Each journal consists of a question concerning the mathematics investigated during the previous week or two, a question about the student's attitude or belief on a particular topic or concept, and a chart to be completed by the student indicating how much time was spent studying mathematics during the previous week. Students like the weekly feedback about their progress and answer the attitude questions in detail. Instructors find the responses informative and easy to assess. At one field site, when asked "What information did you get from the journals?", instructors responded:

> Our classes meet only two days (at most 3) per week. We would take up Journals on Thursday and return them on (the following) Tuesday. We got immediate feedback on

> what the students did and did not understand. Much more than what we got from in-class questions.

In response to the attitude question: "When is the best time for you to do your homework and where?", a married student from that same site replied: "At home, in bed late at night. Nothing else ever happens there anyway!"

Not all students are comfortable or accepting of the reform materials. A typical response from a resistant, usually younger (18-21) student is: "I can't just look at a problem and figure out how to work it if there isn't an example. Just show me how to do it and I'll practice." Based on the reactions of some students reported by those involved in the calculus reform projects, negative reactions should be expected. As James Kaput observed, "you have violated the unwritten social contract. You've changed the role of the student (from passive to active participants), the role of the teacher (from lecturer to guide), and what it is to do mathematics from a student's perspective."

During the past two and one-half years the pilot materials have been used in eleven daytime sections of Introductory Algebra at Harper. Longitudinal data (Spring, 1992–Fall, 1993) for both the pilot and traditional control sections were analyzed to determine what happened in subsequent mathematics courses to students who successfully completed the Introductory Algebra course.[1] The data on the 297 pilot students were compared with data on a control group of 332 students taught using traditional materials and methods. The pilot and non-pilot sections were taught by both full-time and adjunct faculty, with adjuncts teaching more than half of both pilot and control sections.

The study showed no significant differences at the end of the semester between the pilot and control Introductory Algebra groups. Both groups had approximately the same enrollments, withdrawal rates, and completion rates (A–F). One-fourth of each group were students in career programs who satisfied their mathematics requirement in the Introductory Algebra course and did not take any additional mathematics.

However, analysis of successful completion rates (A, B, or C) in the subsequent Intermediate Algebra course indicated significant differences between the pilot and control groups. Statistical results obtained in the study indicated that the number of pilot students who completed the Intermediate course successfully, with a grade of A, B, or C was significantly higher than the number of students from control sections who completed the course successfully.

In addition to looking at the overall success rates for students who completed the Introductory Algebra course, the study also analyzed maintenance of grade to determine whether or not students maintained their grades in the subsequent Intermediate Algebra course. Both groups of students, those in pilot sections and those in the traditional control sections, enrolled in traditional sections of Intermediate Algebra taught by both full-time and adjunct faculty. The percentage of pilot students who maintained or improved their grade in the subsequent course was statistically greater in each of the three successful grade categories (A, B, C) than that of the control students. The follow-up on students who initially attempted the introductory course and who needed to successfully complete a college-level mathematics course indicates a high attrition rate still exists for both groups of students, with only 20% of students who enrolled in the Introductory Algebra pilot course and 13% of the control students successfully completing a college-level course at Harper.

During this next year we plan to continue our study of what happens to students and instructors who participate in a reform curriculum project. We are interested in developmental students' understanding of function and what changes in performance and understanding occur as a result of using technology. We are also interested in finding out if attitudes and beliefs about teaching and learning mathematics change as a result of participation in this project. Some of our field-testers will be working with us in these efforts.

1 The longitudinal study, with summaries of all data is available from Mercedes McGowen, William Rainey Harper College, Palatine, IL. USA.

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