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YEAR 10 STUDENTS' EXPERIENCES OF MATHEMATICS

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This paper describes a discussion of, and participation in, an aspect of the on-going analysis of transcripts of conversations with year ten students. This is one of a number of devices intended to elicit validity for the interpretation of these conversations. One narrow aspect of the interpretation is identified: the mathematical content 'as experienced by the students.' Mathematics content is categorised into five domains of knowledge as articulated by HMI [DES 1985], facts, skills, conceptual structures, general strategies and personal qualities. The discussion focuses upon definitions of these five domains provided by the author and their application to the transcript of a short conversation between two students working on enlargement and scale factors.

Introduction

Transcript data arises from unstructured conversations with students held during their routine mathematics lessons. Every mathematics lesson was attended for two and a half terms and substantive conversations were held with individual or pairs of students lasting from around ten to forty minutes. The main purpose of the research is to explore the goals towards which students work in their mathematics lessons; this is discussed in more detail elsewhere [Goodchild 1993, 1994a,b, 1995a]. The mathematical content and its interpretation by the students forms an important part of the context within which the conversations take place and initial analysis suggests an association between content and students' goals [Goodchild 1995b]. Working definitions of mathematical content are reproduced below.

Definition of mathematical content as experienced by students.

The purpose here is to identify and describe the mathematics which students *may encounter* in their activity, this may be different from the theoretical mathematical content because the application of skills in a particular routine, which is not understood, may conceal the *theoretical* conceptual structure of the mathematics. The following, based on Mathematics from 5 to 16 [DES. 1985] are tentatively offered as working definitions of facts, skills, conceptual structures, general strategies and personal qualities. Suggestions are also made regarding the application of these working definitions indicating how the conversation transcripts may be interpreted to provide evidence that the student has *the opportunity* to encounter these in their activity.

Facts, skills and conceptual structures may be sub-divided into domains of mathematics: number, generalised arithmetic, shape and space, measures, data analysis and display, probability and general aspects of working mathematically.

FACTS (*Know that ...*)

A fact is recorded when it is evident that a student recalls from memory or identifies in some other source, such as the text, a discrete element of knowledge (terms, notation, convention, result). This is not intended to include those 'facts' which relate only to a particular problem or question such as a particular dimension in a diagram. Thus examples of fact are: three times two is six; the decimal equivalent of a half is 0·5; a diameter is a line passing through the centre and joining two points on the circumference of a circle. It is not expected, or necessary, that students would be able to furnish a complete definition for a fact, as for diameter in the preceding example, but they should show evidence of *the opportunity for* using the fact in context. A fact is also recorded where a student is unable to proceed because s/he is unable to recall or identify in some other source a necessary element of knowledge.

SKILLS (*Know how ...*)

A skill is recorded when a student is observed performing some operation to produce a result. Examples of skills in use would be: performing operations with number - with or without a calculator, converting a vulgar fraction to a decimal e.g. the decimal equivalent of a half is given by 1÷2; using a ruler scale to measure a line. More complex skills include the orchestration of a number of elementary skills such as when calculating a new value after a percentage increase. Skills are probably the easiest things to identify because they become evident whenever a student *acts*. A *necessary* skill may also be recorded where a student claims *"I don't know how to ...*

CONCEPTUAL STRUCTURES (*Know why ...*)

Probably the most difficult aspect of mathematics to identify, certainly from these conversation transcripts. The Cockcroft Committee suggests that the presence of a conceptual structure *"is shown by the ability to remedy a memory failure or to adapt a procedure to a new situation"* [DES. 1982 para. 240], this is reasonable but too restrictive in the present exercise. It is probably easiest to identify where a student's conceptual structure is not well formed due to their insistence on the application of some (inappropriate) fact or skill. Concepts may be implicit in the activity and this may be recorded, thus in graphing linear relations a student has the *opportunity* to encounter *letter as variable* but when solving simple equations with one unknown a student encounters *letter as specific unknown*.

At this stage no distinction (in terms of ascribing value) is made between conventional understanding (as held by the mathematical community) and student's understanding (which would be described as error or misconception by the mathematical community), both are recorded as conceptual structures which the student has the opportunity to form, develop or apply.

GENERAL STRATEGIES

These are identified when it is apparent that a student is using a strategy, process or routine to solve a problem which is not specific to a particular topic. Five broad categories are identified: (i) general problem solving strategies e.g. trial and improvement; (ii) strategies related to the application of skills and routine procedures e.g. following an example; (iii) strategies of reflection e.g. checking working and interpretation of answers: (iv) strategies for the use of text e.g. reading carefully; (v) coping strategies i.e. means employed by students to cope with problems, implementation of coping strategies enables the student to 'avoid' engaging with the mathematics of the task, e.g. copying.

PERSONAL QUALITIES

Evidence from the conversations which reveals characteristics of the student's attitude and feelings. Five sub categories are identified: (i) a desire to avoid work or complexity; (ii) a readiness to engage; (iii) determination to succeed; (iv) understanding the nature of the activity and their part in it; (v) acceptance of an interpersonal dimension (e.g. the value of collaboration) in the process and product of their work.

Discussion

Possible areas of ambiguity and weakness in the above working definitions were identified. In particular it was suggested that the example of *two times three* as a fact was unhelpful because underlying the use of that fact was a conceptual structure which identified this operation as appropriate to a given context. Thus an alternative example was provided: a student may know as fact that 25% is equivalent to 0.25 and be able to use this in a routine skill to work out 25% of a quantity, but there need not be any underlying conceptual structure which relates 25% to 0.25. It was also argued that there must be a conceptual structure of some form underlying the routine application of skills, such as the use of a scale to measure length. The point is well made, in these observations, that a student's response, oral or written, may be evidence of mathematical knowledge in more than one of the domains listed.

It was also revealed that a further 'qualifier' was required to direct the application of the categories, so the first sentence: *The purpose here is to identify and describe the mathematics which students may encounter in their activity* .... should also indicate that this *encounter* needs to be possible *within the interpretation the student is making.*

The application and subsequent discussion of these categories of mathematical content to the transcript provided for this purpose did not conflict with the preliminary analysis of the 130 conversations collected during the course of the study. The analysis of these conversations reveals a surprising fragmentation and lack of coherence in the students' experience of mathematics. In the analysis, each element of fact etc. is identified and asociated with the topics in which it arises. This process reveals a very large proportion of elements of facts, skills, conceptual structures arise in one topic only. Thus the accepted notion of mathematics consisting of a rich web of interconnections is not the experience of the students. The table below shows the results of the preliminary analysis.

Number of topics in which elements arise	**Facts** (Total - 146 elements) Number of elements	**Skills** (Total - 174 elements) Number of elements	**Conceptual Structures** (Total - 142 elements) Number of elements	**General Strategies** (Total - 53 elements) Number of elements	**Personal Qualities** (Total - 13 elements) Number of elements
1	130 (89%)	148 (85%)	131 (92%)	41 (77%)	10 (77%)
2	10 (7%)	11 (6%)	10 (7%)	7 (13%)	3 (23%)
3	4	5	1	2	
4		2		1	
5		1		1	
6		1			
8		1			
9	1	2		1	
10	1				
11		1			
13		1			
17		1			

Post script

I am grateful to Alan Bell who, after the session, suggested that an alternative framework for this analysis may be more appropriate, observing that skills and general strategies are about what one *does* in time, and thus the 'connections' between elements would occur in a linear fashion, whereas facts and conceptual structures, belonging as they do to mental schema, are more likely to be richly interconnected.

References

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