

## CHARACTERISTICS OF UNDERSTANDING

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*This paper builds on a previous one, presented at AMET/BSRLM. We have continued to work on the meaning of understanding since then, making use of the comments of others and looking further at the literature. We have recently come to see the core of our research as attempting to look at the learner as if from the inside - as if we were the learners themselves. However, as teachers and observers of the learning of others, we have been attempting to link our thinking about the internal processes of the learner with their possible, observable external manifestations. It is the nature of, and the distinction, between the internal and external characteristics together with an examination of them as consequences of our theory which we explore in this paper.*

### **The Theory and the Definition of Understanding.**

Our theory, which has been explained more fully elsewhere (Duffin and Simpson, 1994b) starts from the premise that learning is the act of building internal mental structures to cope with, and in response to, experience. Our aim has been to classify the types of experience which a learner may meet, depending on the way in which the experience fits with their existing internal mental structures and to indicate what possible responses, in terms of modified internal mental structures, these experiences may cause.

We call our three types of experience *natural*, *conflicting* and *alien*. In brief: a natural experience fits with the current internal structures and, in doing so, strengthens them. A conflicting experience jars with expectations and highlights limitations, contradictions or unexpected connections between internal mental structures, leading the learner to destroy their current way of thinking, limit its scope or merge two previously distinct ways of thinking. An alien experience is one that fails to fit with the learner's way of thinking and leads the learner to ignore or avoid the experience or to absorb it as a new, isolated structure which, through further natural, conflicting and alien experiences may become more fully integrated into their thinking.

With this theory we analysed a number of different incidents from our own teaching and learning in some detail (e.g. Duffin and Simpson, 1995). We were aware,

however, that such a detailed approach was inappropriate for general use, for a teacher trying to make sense of their pupils' learning in the classroom. We felt that there was a need to build up a more easily applicable theory. In doing so, we decided to concentrate on giving a clear meaning to the term 'understanding' because we frequently found ourselves wanting to use the term, but without having a clear conception of its meaning within the new framework for thinking about learning which our theory had brought us.

Starting from our own feelings about understanding, we looked at our theory and asked ourselves whether anything in it could account for those feelings. This led us to a definition of understanding in terms of internal mental structures.

*We define a learner's understanding as their awareness of connections between their internal mental structures.*

In Sierpinski's terms (Sierpinski, 1990), we see understanding as a *state* while the act of *coming to understand* is the process of building and becoming aware of these connections through responding to natural, conflicting and alien experiences. We do not, however, see understanding as an 'all-or-nothing' state, we consider that more connections indicates, in some sense, more understanding. From Nickerson (1985) we also see understanding as having two dimensions - breadth and depth - with more direct connections from a concept to other concept as more breadth and more indirect connections to still further connected concepts as more depth.

### **Three perspectives on Understanding.**

Up to this point what we have written has been very theoretical - providing a *technical* meaning of understanding in terms of our underlying theory of learning. We are also aware, however, that there are direct personal meanings for understanding and also practical ways in which we believe we can begin to observe understanding. Thus we have three perspectives on understanding to connect, the theoretical, the internal and the external and we see three types of sentence which we feel will help us to develop those links:

Consequences of the theory:

If *a learner* is aware of connections then *they* ...

Internal characteristics

If *I* understand then *I* ...

External characteristics (indicators)

If *my student* understands then *I* can see ...

We want to postulate that the internal characteristics of understanding which we have identified are just those which we can derive as consequences of our theory and that, in turn, those observable, external characteristics (which we call *indicators*) can be seen as the external manifestations of those internal characteristics.

In looking at the link between internal and external characteristics, we are looking at teachers as builders of models of their pupils' understanding, searching out experiences from their students in order to enhance or modify their internal mental structures concerning them.

In putting the last two of these three sentences to the audience when we presented the ideas in this paper, we found a strong agreement between their lists of internal characteristics and indicators and the list which we had developed. They also raised issues about the feelings learners associate with understanding, expanding on our suggestions in Duffin and Simpson (1994a)

### **Internal Characteristics**

We suggested that, if we personally feel we understand something, then we are able to explain it, we can put it into simpler terms. This can also be seen as a consequence of our theory because, in explaining, one uses internal connections between the concept to be explained and the simpler terms in which one wishes to cast it. We also note that our theory suggests that with a deeper understanding (that is, having more connections that lead to further connected structures) one might be able to provide a deeper explanation, breaking more of the ideas down into simpler ones and this too accords with our experience.

We also considered that if we understand something then we don't need to remember it in full detail, but that we can reconstruct it when we need it. Our theory suggests

that being aware of connections would allow a learner to draw in knowledge related to that which needs to be reconstructed and to find associated ideas. So we see the ability to reconstruct as both a consequence of our theory and an internal characteristic.

This seems to be the case with another characteristic - the ability to see concepts in unfamiliar contexts. This can be seen as a consequence of our definition since, using connections between the concept and the internal mental structures concerning the context, the learner can abstract those parts of the context that relate to the internal form of their idea.

Our final internal characteristic we had missed initially. But, on reading Nickerson (1985) we were suddenly saw one of his notions – the ability to derive consequences and predict what might happen in different circumstances – as another internal characteristic. From the perspective of our theory we suggest that if one is aware of connections between a concept and other related ideas, by looking at how they work, one can begin to use those connections to say what might happen if some part of the concept is changed.

### **Indicators of Understanding**

In addition to connecting our theory to our perceived internal characteristics, however, one of our aims was to see how teachers might attempt to assess the understanding of their pupils and how teachers' observations relate to the internal characteristic we had identified. Again, we found that the audience at this paper's presentation gave very similar external characteristics to our own.

We suggest that one of the simplest methods that teachers use to determine understanding is to see if their pupils can *do*. That is, the teachers ask them to perform a task. This may be seen as an attempt to see if a pupil can reconstruct the concept or process.

Teachers also ask their pupils to put something into their own words, to explain it to the teacher or to their peers. They also set problems for their pupils which will be put in a slightly different way or in a different setting. These are clearly attempts to identify the external manifestations of the internal characteristics which are the ability to explain and the ability to see in other contexts. Teachers may also ask their pupils

to answer 'what if...?' questions in an attempt to observe whether they can derive consequences.

However, while this seems to suggest a strong match between the internal characteristics of understanding and the (external) indicators, we have become aware that there can be considerable difficulties in determining if an indicator is reliable. For example, as an observer, can we really be sure that a pupil is reconstructing rather than merely reproducing? This was the basis of the difference between two incidents we discussed in an earlier paper (Duffin and Simpson, 1994a). Similarly, can we be certain that a pupil's explanation is actually their own and not the recitation of someone else's words?

In particular we feel that the ability to *do* is a very poor indicator - we are sure that anyone working in mathematics education is aware of instances when a pupil has been able to do something, but there is no indication that they understand and, perhaps more importantly, we may be able to think of things which we can do but which we are aware we do not understand.

However, it appears that seeing if pupils can answer 'what if...' questions or seeing if they can solve unfamiliar problems may be a good indicator of understanding, since recalling or reciting are not available in a situation which is truly unfamiliar.

### **Summary**

We have seen, then that there are three perspectives from which we wish to look at understanding - the theoretical, the internal and the external. We have suggested that internal characteristics can be seen as consequences of our theory and definition of understanding and that, in building their own model of their pupils' understanding, there are a variety of indicators which a teacher may use, with differing reliability.

In what we have said, we have deliberately separated the indicators to highlight their connections to internal characteristics. Teachers, though, are not restricted to looking for one indicator at a time, we try a variety of methods and observe a variety of indicators which act as natural, conflicting and alien experiences in the development of our internal models of our students.

### References

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