INVESTIGATING THE MATHEMATICS LEARNING OF STUDENT TEACHERS: EXPLORATIONS AND DISCOVERIES?

Paul Blanc

King Alfred's College (Winchester), Institute of Education, London.

This paper describes a *single experiment from early stages of work in progress. It examines one student's work in depth following the analysis of* a *class set of solutions of first year teacher trainees and mathematics students for* a *well-known investigation.*

Backeround to the research.

Changes to the mathematics curriculum that followed from the Cockcroft Report (Cockcroft 82) led to "Real" Problem-Solving and Investigations becoming part of the official content of mathematics. The inclusion of such activities as assessed coursework for GCSE has ensured their being common occurrences in UK schools, and influenced the ways in which these investigations are carried out. In the mathematics education literature and particularly the journals closest to the classroom teacher there has recently been a debate about the effectiveness of such mathematical investigations in compulsory education mathematics classrooms. Hewitt (Hewitt 92) has questioned whether the diversity and richness of the mathematics curriculum was being reduced to spotting patterns from tables. In response Andrews shows concern that if those who were attempting change "see their efforts undermined by such criticism, is there not a risk that they might revert to the didactic modes of operation that characterise all that is disappointing in mathematics education, but which both teachers and pupils seem to find comfortable and unthreatening?" (Andrews 93). Wells in a contentious pamphlet introduces the notion of Data-Pattern-Generalisation (DPG) as a general mechanistic method of solving problems said to have little or very limited mathematical value (Wells 93). William (William 93) claims that "student's perceptions of the didactical contract was that their task is to find THE RULE", adding "What I find particularly saddening about the DPG paradigm is that it reduces mathematics to an empirical discipline." Relating this to Teacher Education, Wells (op cit) suggests that in some courses "students are expected to do investigations, ... , thus, being inducted into false and very limited ideas of mathematics." The overall thrust of this material appears to be that the potential positive advance that pupils explore mathematics at their own level has been seriously undermined by the algorithmic and mechanical nature of the approaches adopted.

The lareer study.

The overall aims of my study are to examine ways in which beginning teachers approach a range of problems of an investigative nature and specifically to identify and analyse the different influences that affect these solution strategies. Aspects to be considered include: school experience, prior subject knowledge, forms of representation used in solution, specific solution "algorithms" (or problem solving methods) known to the students, and implicit or explicit structure put forward by tutors. With regard to student goals, interventions, mechanistic exploration and heuristics one of the main influences on this study is the French "didactique" of Guy Brousseau, (Brousseau 96) who claims that the interactions in a mathematics classroom can be viewed in terms of a (didactic) contract. Brousseau defines this is as set of rules which organise the relationships between the content taught, the pupils and the teacher within the mathematics classroom. Pupils enter any new problem with reference to a contract which; means that proposed problem is a legitimate activity, guarantees that a solution exists and so on. The paradox of this didactic contract is that everything the teacher attempts in order to make the student produce the behaviour she expects, tends to deprive the student of the necessary conditions for the understanding and the learning of the target notion. In this model: Teaching is not the communication of knowledge but the *devolution* of a good problem (an a-didactical situation). Learning is the students adaptation to this situation. In such adidactical situations the teacher: provokes adaptation, chooses problems, and holds back from giving information; the student: accepts problem as her own and interacts with it. The teacher decides how much she will offer in terms of: information, questions, methods, heuristics and so on (Brousseau 86 translation by Balacheff 94).

My setting is teacher education at King Alfreds' College, Winchester and my sample first year students taking modules offered by the department of mathematics. The current cohort is made up as in the table below.

	BA (QTS) Primary		**BSc Modular**		**Total**
	Male	Female	Male	Female	
Mature	0	3	1	5	9
School Leavers	3	8	1	6	17
Totals	**14**		**13**		**27**

The specific problems/investigations which I am using in the pilot stages of my research have been taken from, or incorporated into, the teacher education programme in as natural a way as possible. Students tackle the investigations within the normal course hours, the content area covered by any of the experimental sessions being closely related to the topic of the day. There are a wide variety of tasks and differing teaching styles experienced during the modules. The modules are Discovering Mathematics (including Pascal's Triangle, Fractals, Graph Theory, Fibonacci, Max Box) and its successor module Exploring Mathematics where more specific attention is given to content areas, Number, Algebra, Geometry and Calculus.

The first pilot experiment.

In order to examine some work before the course influenced their practice, students were given the well known task below to work on in pairs during my first session teaching them.

> **How many diagonals does a polygon have?**
> **Investigate.**
>
> Please produce a written solution to the above problem.
> Please make sure that you explain your solution.
> Please don't erase working or attempts that you reject as
> I'm interested in all your work.

For readers unfamiliar with this problem one possible solution is as follows:

Let 11 be the llumber of sides of the polygon. Then the number of diagonals = $\frac{n(n-3)}{2}$

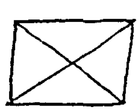
Data was collected by; analysis of student scripts (photocopied), taping of the session (dictaphone), notes made during the session and immediately afterwards. All students completed a questionnaire on their educational and mathematical background. Five students' (offering a range of background and experience) were selected for more detailed analysis and interviewed about their scripts (taped and transcribed).

In the remainder of this article I will examine the work of one student, who I will call Wendy, a mature student who had done GCSE investigations recently, enjoyed investigative work and felt that she had the greatest success in work of this type. Due to limitations of space I will offer a snapshot of my analysis.

Tables ..... and other Representations

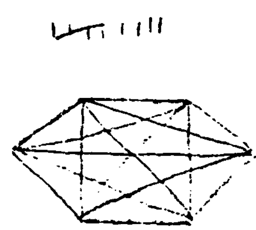
Wendy's text begins by making sense of the question, she states her goal to produce a formula relating number of sides to number of diagonals. A sequence of examples follows with some

interesting annotations.



4 sides
2 diagonals

$\frac{n}{2}$

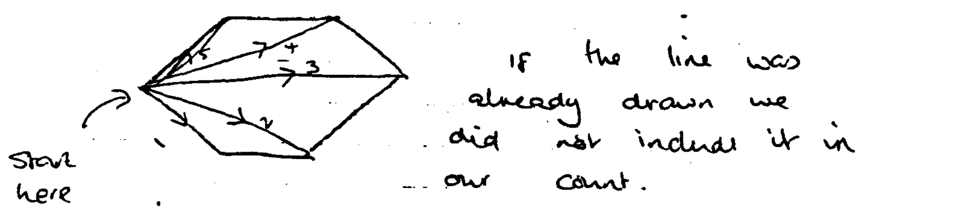


6 sides
~~5~~9 diagonals

$\frac{n}{}$

Note the number of different representations: diagram, tally, *nl2,* and list. Next Wendy produces a 2 column table with headings sides and diagonals. When asked *"why did you decide to do that?"* Wendy replied: *"to look for a relationship between the number in the diagonals column and the number in the sides column."* When asked why now? *"because .. I think, we'd counted the diagonals in quite a few shapes and we were perhaps getting quite disorganised and I wanted to get some order into it."* I wanted to know if this "organisation" had been taught. Wendy was very resistant to this possibility: *"No, not that I remember, ... I don't ~ remember anyone saying stop and think about it"*

Countin2 strate2V (route to a rule via algebra)



Wendy decides to "go around the shape to check each route" a method of counting the diagonals. For the benefit of the reader she demonstrates this for part of a six-sided figure. The method adopted is: count all diagonals from one vertex then move on to the next vertex. No diagonal is counted twice. Here she states: "each time we changed vertex the number of lines to be drawn decreased this made us think of a series formula." Gill her partner, produces 6 separate diagrams showing the diagonals which leave each point, for an octagon. This gives 5+5+4+3+2+1 as the solution. This is quickly generalised as a recursive formula. Although they sum the series correctly (with some help), the manipulation of algebraic fractions (in adding the largest number twice) produces an erroneous formula that works for 3 sides but goes wrong for 4.

Conviction (how do you know that you are right)

This error prompted me to ask what would have convinced Wendy her final solution was right. *PB:*

~vhen you'd done 6 *sides would, ... you'd have stopped then*
W: if I tllOuRht it was ... if it felt right I'd probably stop yeah
PB: when you say felt right ... so if it had workedfor the examples would that have been enough W: it's hard to know If I could have seen, cos sometimes you work out a formula and when you get there you know exactly where it came from and you know before you do it that the formula's going to work and then I'd do one or two tests and if they worked then yes I'd stop there but if I wasn't quite sure of it had come together out of a bit of luck and I wasn't sure where it had come form then I'd probably want to do more
P B: and in this case was it something you felt quite sure about or was it something that you felt... a bit of luck .. I think you said
W: no I think it was something when we actually got there it was something that I felt quite sure about because I was sure that we'd got the series formula bit right because we could see the lines you know there were different amounts of lines from each vertex I could see that it

was definitely going down by one so I could see that it was definitely an arithmetic progression and so once I'd got that formula straight I was confident that that was true.

Observations from this one case.

The tables and data collection appear to have an organisational or structural role. They do not, in my opinion, come from some mechanical behaviour to do with pattern spotting. I am struck by the many different representational strategies used. These are not just isolated jottings but appear to be intrinsically linked to the overall solution: sometimes they are links towards algebra, sometimes they are checking devices, sometimes they are just guesses. The counting process Wendy and Gill use is important in their formulation of a solution. Another critical aspect is that algebra (which is often seen as the end product) seems more than this. It contributes to the series summation and when the formula fails the students are sure that it is not their conceptual solution that is wrong but their fluency with algebra.

Some Questions

If there is a more "elegant" solution? How can you get people to see this visually? Is Wendy's solution any less geometrically visual? Did the use of a table prevent Wendy seeing a direct solution rather than a recursive one? The counting/drawing method here is a crucial element of solution and conviction. It would seem that it retains context. Is retaining contextual material important for successful rule generation? If so, in what way (if any) does this help rule justification?

I have done further experiments with similar "pattern spotting" style activities in different contexts including one using the spreadsheet, and another with Cabri-Geometre as a mediator. I am now developing a framework of richer descriptive tools and methods of analysis for such investigative/problem solving activity.

References

Andrews, P. (1993) Train Spotters Have Feelings Too, *Mathematics Teaching 142,20-22*.

Brousseau, G. (1986) Foundations and Methods of Didactique, unpublished translation of Fondations et methodes de la didactique des mathematiques. *Recherches en didactique des mathematiques*, 7/2, p33-115.

Cockcroft, W.H. (1982) Mathematics Counts, Report of the Committee of Enquiry into the teaching of Mathematics in Schools. HMSO, London.

Hewitt, D. (1992) Train Spotters' Paradise, *Mathematics Teaching 140, 6-8.*

Wells, D. (1993) *Problem Solving and Investigations,* Rain Press, Bristol.

William, D. (1993) Paradise Postponed? *Mathematics Teaching* 144, *20-22.*