Numerical Strategies of 'Low Attainers' and 'Beginning Algebra'

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Area of Research

The main aim of the study is to investigate the developing understanding of algebraic concepts of a group of low attaining pupils as they work within a Logo environment.

Objectives

- To study the development of algebraic understanding of a group of 13/14 year-old pupils who are low attainers in Mathematics, as they pursue a programme of study within a Logo environment.
- To study this development within two fields - generality and functionality.
- To study the way in which pupils make use of their Logo experience in non-Logo contexts.
- To investigate the relationship between the development of algebraic understanding and knowledge of arithmetic, with a particular focus on pupils' informal methods.

Background

In the study the view taken of algebra is that it is a succinct language which facilitates the representation and communication of generality and functionality within mathematical structures.

Further, low attainment is viewed as a measured state of mathematical achievement which does not necessarily give a view of mathematical potential or reasons for the apparent level of achievement.

As a result of a pilot study which was conducted in order to develop an appropriate way of working with the pupils in the main study it was felt that not enough information had been collected about the number competence of the pupils in the study.
The main study will be conducted as per the schedule illustrated below:

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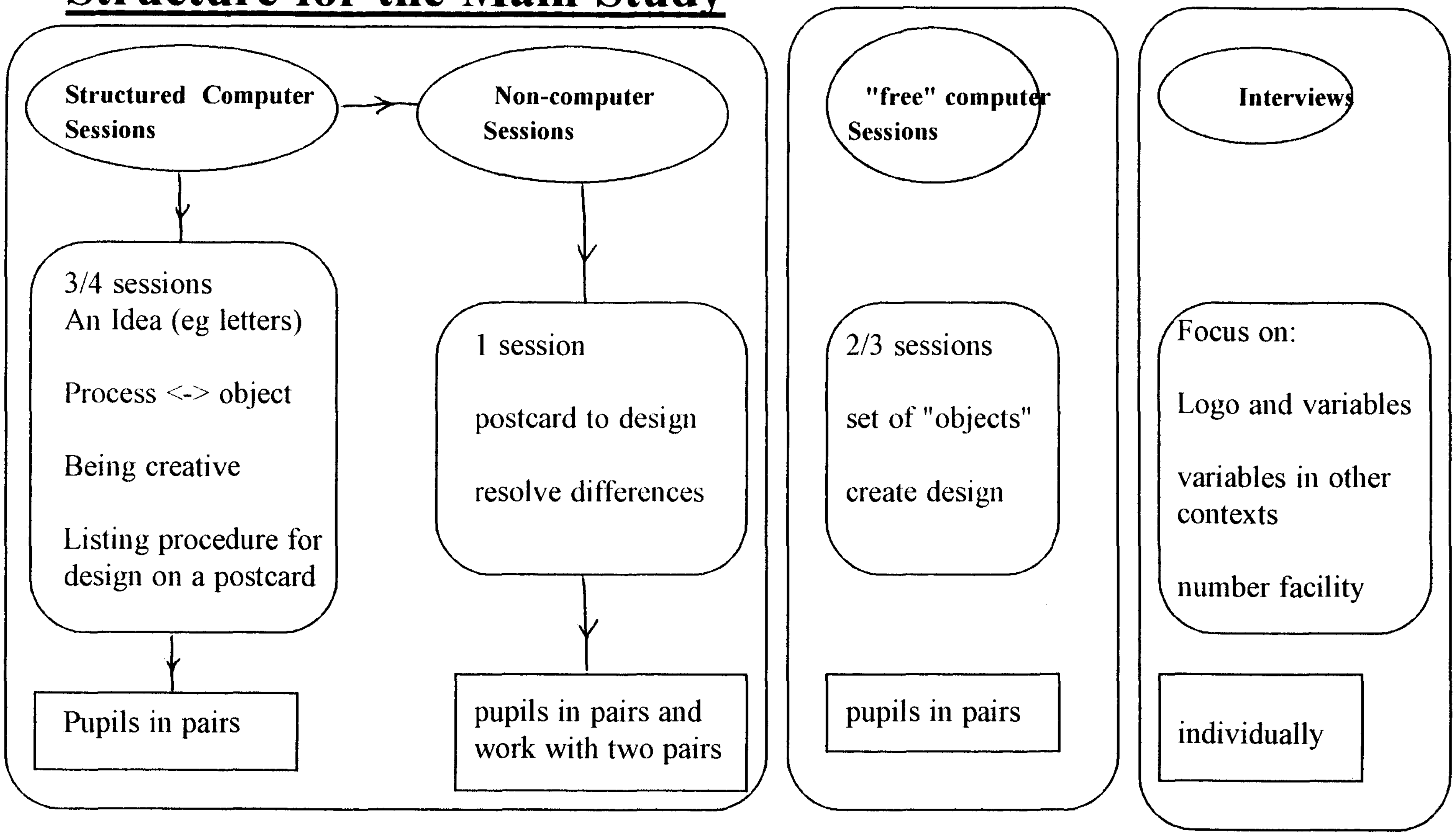
Structure for the Main Study



This is the proposed structure for each block of work which will last 5/6 sessions
Thus during the year there will be about 5/6 blocks of work.
The re will be interviews at the beginning middle and end of the study
The "free" computer sessions will be used if necessary at specific points of the study

Number Competence

In order to gain information about the pupils who will be involved in the main study, (Caroline, Charlotte, Chris, Katie, Kelly, Laura, StuartP, StuartR) initial taskbased interviews were conducted with each pupil individually. The questions in the interview all involved number calculations. Some were simply about calculations others were word problems which required number calculation in order to solve them. The aim was to gain an understanding of the way in which the pupils approached numerical problems and the methods they used to complete numerical calculations both orally and on paper.

The initial interview was conducted with each pupil individually. The objectives of the interview were:

1. To gain an understanding of a pupil's facility to work with numbers.
2. To gain an understanding of a pupil's competence in numerical manipulation.
3. To gain an understanding of a pupil's facility to interpret word problems.

The interview consists of three parts:

A Exploring pupil's number concepts and number competence.

B Word Problems

C Using letters in mathematical contexts

Some Thoughts on the initial interviews.

There are some interesting issues arising out of these interviews. They relate to the differences/similarities between the way pupils perform orally and on paper. In most cases, presumably because it was in a school context, the pupils initially tended to try a paper and pencil method. They needed to be encouraged to use non-paper and pencil methods.

Problems involving Addition

Out of the 8 pupils that were interviewed, all of them tended to use a pencil and paper method initially. But when asked to try the sums in their heads without pencil and paper, three of them used quite different methods and 5 of them attempted to follow the written algorithm mentally. For the three pupils who used different methods mentally there were some consistent observations.

The pupils worked from the left of a sum i.e. with the digit of the highest place-value They held on to the place-value of the digit i.e. in 27 the 2 was spoken of as 20 not as 2. The numbers were sometimes partitioned in different ways, so that 15+26 could be done as 10+20+11 or as 10+20+5+5+1. Sometimes (especially with Charlotte) the partitioning included negative numbers e.g. 23+48 was done as 25+50-2-2.

Those pupils who maintained the same method orally as on paper seemed to depend entirely on unreflective recall in order to work through a sum. A typical reponse for 23+48 would be:

3 add 8 *is 11*
put 1 *down there (in units column) 23 put* 1
down there (under the tens column) 48
2 add 4 *is* 6 *add* 1 *is* 7 71
put 7 *there (in tens column)* *1*
71

or

3 add 8 *is 11*
put 11 *down there (in tens and units column)23*
2add4is6 *48*
put 6 *there (next to the* 11) 611
611

In these methods the place value of a digit was implicit by position as opposed to being explicit in the some of the alternative oral methods. It would also seem that the pupils who tried to follow a pencil and paper algorithm in their head were having to perform quite difficult mental memory tasks. The following examples illustrate the oral methods of some of the pupils.

Extract 1 - Adding 22 and 49

P *I started with the* 4 *and added the* 2 *then it equalled up to sixty. Then I added the* 9 *and the* 2 *and I got the number*

T. *What was* 9 *add 2*

P. *eleven*

T. *Then what did you do?*

P. *Then I added an extra* 1 *to that and that was how I got the sum.*

Extract 2 - Adding 37 and 15

P: 3 *add* 1 *gives forty add the* 5 *and the* 5 *to get* 50 *and then add the 2*

Extract 3 - Adding 22 and 49

P *Fifty plus twenty is seventy. Then add* 3. *Seventy-three. Oh dear .... I'm not sure what to do with the* 2 *and the 1.*

Problems involving Subtraction

Of the 8 pupils interviewed all tried paper and pencil methods first. But only two of them seemed to have different oral strategies to those used on paper although the methods weren't always carried out accurately.

Charlotte used a similar method to that which she used for addition in that for the sum 46-29 she first of all calculated 50-30 and then considered what to do with the 1 and the 4 that were left.

StuartR. used a variation of standard paper and pencil algorithm, in that for the sum 41-13 he did:

40-10=30

1-3 you can't do

Borrow from the 3 and make it 2

11-3 is 8

So it's 28

All the other pupils either used the same method - the standard paper and pencil algorithm - when working orally or on paper, or simply refused to try to work the sums out orally. Standard errors in the paper and pencil method were pupils would simply work in each column and take the smallest number from the largest in each column, or they would write the sum down with the smaller number on the top, or the numbers would be written in the wrong column. For example:

	0 1
35	~~1~~00
83-	25 -
52	25

Often sums were written down in the order in which they appeared in the question. One pupil who performed accurately on paper tried the same method orally even with three digit numbers and thus was holding in her head a large amount of information. Some of the oral methods were as follows.

Extract 1 - 15-6

Take 5 is ten

Take 1 is nine.

Extract 2 - 41-13

Pupil counted up from 13 on fingers and mentally recorded the number of tens.

Extract 3 - 41-13

40 take 10 is 30

1 take 3 you can't do

Borrow from the 3 and make it 2

11-3 is 8

So it's 28

It is more difficult to draw conclusions from this information but what can be said is that once again in oral methods pupils start from the left with the highest value digit and the place value is explicit in the progress of the sum. Also using the paper and pencil method orally requires the holding of a substantial amount of information in their memories. The confusion between oral and written methods is illustrated in the extract below:

$$\begin{array}{r} 27 \\ \underline{23-} \end{array}$$

P: *That's 14*

T: *How did you do that?*

P: *Took the* 5 *from that, added those two together to make 10*

(This would appear to be taking the 5 from the 7 and adding this to the 2+3)

T: *You took the* 5 *from the ..*

P: 7

T: *Yes*

P: I *did it to the* 2 *and the* 3 *and then took the* 2 (which was left from the 7) *and the* 2 *from that* (the 27) *and added them together which gave me 14.*

T: *OK* so 27 *plus* 23 *is 14*

P: *Yes.*

T: So *can* I *just check you're starting with* 27 *and* 23. *You took the* 2 *from the 7 ...*

P: *Oh, it's taking away. Oh yeah* 14 *works out.*

Summarising the addition and subtraction processes observed orally and on paper a number of points could be made:

1 There are similarities between the findings from this work and that of Nunes *et al* (1993) in their project in Brazil, with street children, in that there was evidence with our pupils that they too had oral methods of working which required a different form of representation to the pencil and paper methods which they were often taught in school and so the school methods seemed at times to be in conflict with their own individually constructed mathematical understanding.

2 The oral methods used often hold on to the place value of the digits in a number orally whereas the standard algorithm recognises place-value by the nature of the position of the digit on the paper.

3 The oral methods used involve what could be called the combination of like terms in that the pupils first add the hundreds and then the tens and then the ones.

4 Methods used are strategies rather than algorithms in that the strategy used involved partitioning of numbers but not in the same way each time. The paper algorithm requires strict adherence to a set of rules.

5 To use Vergnaud's (1982) terminology the pupils were demonstrating a number of theorems in action as they worked when they worked without pencil and paper. These again are very similar to those that Nunes et al found with the Brazilian children. They were:

- A number can be partitioned without it losing its total value. This was evident not just in terms of tens and units but in other partitions also e.g. 27 as 25+2
- The addition and subtraction of numbers can be carried out on the partitioned parts without affecting the final result. This did result in some problems when the partitioning involved subtraction

M ul tiplicationlDivision

In multiplication sums only one of the pupils used oral methods which were different to the paper and pencil methods. The oral method that was used by StuartR involved partitioning the number using the "nearest 5" and then using repeated addition. All the other pupils would only try to do the standard paper and pencil method. This was rarely completed correctly. Division questions caused similar problems in that the pupils would not attempt them. The tentative summary analysis produced below is based on the addition and subtraction work of the pupils.

Oral and Written Comparisons for Numerical operations.

Oral	Written
partitioning of numbers, varied groupings	partitioning by position, fixed grouping
Development of theories in action	unreflected recall, no theories in action
work from large to small and preserve relative value of digits	work from small to large and relative value is represented by relative position
a personal learning experience leading to construction of personally meaningful concepts?	an externally imposed experience which is designed to lead to technical competence
learning is context-based and happens through observation and immitation?	learning is through verbal explanations and memorisation of procedures
Meaning is preserved throughout an operation	methods rely on procedures that distance themselves from meaning
the range of numbers within which pupil work is restricted.	allows for a greater range of situations to be worked with.

An overall summary of the results of the task-based interviews would seem to be: 1 There are areas of numerical work within which pupils experience blockages.

2 Choice of operation and performance of operation are at different stages of development.

3 Some pupils do have oral methods of working which are different from standard paper and pencil methods and require a different form of representation.

4 With paper and pencil methods pupils do not have a feeling for the correctness of an answer whereas oral methods may help to develop this feeling.

S Oral methods seem to help pupils to develop 'theorems in action' whereas the standard paper and pencil methods seem to rely entirely on unreflective recall.

It could be that the lack of development of oral methods and thus these theorems in action could have an adverse effect on the development of algebraic concepts which in some ways could be seen as the representation of these theorems.

References:

Nunes, T. et al (1993) *Street Mathematics and School mathematics,* Cambridge, Cambridge University Press.

Vergnaud, G. (1982) 'Cognitive and developmental Psychology and research in Mathematics Education: some theoretical issues', in *For the Learning of Mathematics,* Vol. 3 No.2.