Mathematics Mentors' Pedagogical Content Knowledge: some suggestions for structure

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The 'mathematics mentor' is the school teacher tutor for student teachers within school-based Initial Teacher Education (ITE). My anecdotal and recorded observations of mentor-student interaction indicated that discussing points of mathematics and mathematical pedagogy were rare, (whereas discussion of general issues, like classroom management, were frequent). Student teachers need to learn strategies for teaching mathematics in particular, as well as children in general. As the students are based in school, the mentors have a key role in developing this learning; what then, is the nature of the mentors' knowledge and how might they express this knowledge?

While I do not attempt an exhaustive analysis of the reasons for why the mentors do not prioritise the teaching of mathematical pedagogy, I suggest there are at least the following considerations:

(i) The experience of many of us who have worked with novice teachers is that they have a urgent concern with getting 'good order' in their classrooms, so that working on management issues can easily exhaust the limited time available for tutorials with the mentor in school.

(ii) The register (see, for example Pimm 1987) of mathematical pedagogy is not well developed in schools, so it does not naturally bubble into conversation.

My work with mentors indicates that they have varied and definite views on the subject of mathematical pedagogy. I suggest that structuring this knowledge by naming certain aspects of it would be helpful in raising its profile in mathematics secondary ITE. In other words, how to structure pedagogical mathematical knowledge so that mentors become more aware of this knowledge and are better able to develop it in their students? I aim to offer, below, constructs that are distinctive to mathematics and that are helpful in the business of teaching novice mathematics teachers.

The concept of 'pedagogical mathematical knowledge' is derived from Shulman's general concept of "pedagogical content knowledge":

> *That special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding.*
>
> *The blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organised, represented, and adapted to the diverse interests and abilities of learners and presented for instruction.*

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Shulman (1987)

One of the key features of school-based ITE is that the student teacher should have access to the teacher's craft knowledge (see, for example, Haggar *et al1993).* While a teacher's craft knowledge incudes many generic aspects of teaching, it also includes aspects of mathematical pedagogy; how to teach this topic to these children at this time.

In order to work on mathematical pedagogical knowledge, I believe that it is useful to have examples of one's own practice to mind in order to examine how a proposed structure might fit with experience. For example, I recently taught calculus class to (adult) college students where the topic was *e* (the 2.718 .. variety!). My 'pedagogical' knowledge was realised both in the forward planning that I did and in my momentary decision making as the lessons unfolded. I believe that in both of these aspects certain themes are discernable:

(i) I viewed the mathematical topic on occasion *'holistically'* and at other times' *atomistically'* (focusing in on detail). For example, I chose to link in power series and complex numbers and I wanted to work on the detail of first -principles differentiation.

(ii) I worked on the students making meaning for themselves through *'negotiation"* in discussion and adapting their ideas and I checked that these meanings were developing to the *'precise'* received mathematical concept. For example, I asked them to share what they knew / remembered of the exponential function and then tried to make precise the concepts that they had offered, e.g. we looked at the student offering "It's the thingy that's the same when you differentiate" first graphically then analytically, so hopefully they now know in what sense the quoted offering is true.

(iii) I gave scope for *'creative'* individual expression, but also wanted to facilitate *'mechanical'* abilities (e.g. ability to employ algorithms where appropriate). For example, there were occasions when I asked the students to investigate properties of a procedure or of a formula, the results of which fed into the class's understanding of the exponential function and there were occasions when I asked them to calculate derivatives and integrals of functions that included exponentials.

These considerations lead me to suggest that we could consider these three dimensions of practice:

The Holistic - Atomistic Tension: *an aspect of viewing mathematics*
The Negotiation - Precision Tension: *an aspect of explaining mathematics*
The Mechanical- Creative Tension: *an aspect of doing mathematics*

(To the question "are they are parallel or orthogonal?", my reply is that they form a manifold in the space of dimensions of pedagogical content knowledge!

Other subject disciplines will have analogous categories, my contention is that to learn how to teach any subject it is helpful to have some 'dimensions of practice' in order to help focus attention and raise possibilities for choice in future teaching situations. In particular, in order to help others learn to teach, the mentor needs to be able to recognise the legitimacy of other views and practices and to share her/his own craft knowledge. To address the 'width' of variety together with the 'depth' of personal conviction, we need a language to express these, or similar, 'tensions' perceptible in the practice of teaching mathematics. Awareness of these 'tensions' might facilitate both a wider choice for the student and a more informed articulation of the mentor's position.

The tensions noted above were developed from two principle sources:

- my reflection on my previous role as a mathematics mentor and my current role as an HEI tutor who works closely with mathematics mentors in partnership schools
- interviews I had with five of the mathematics mentors that I recorded towards the end of the 93/94 academic year. It is to some quotes from the mentors that I should now like to turn.

The following pair of quotes can give an idea of a range of practice within the 'holistic-atomistic'tension:

> *A: I deliberately do ('break it down') because when I started teaching maths I had to think very carefully and I always realised myself that it had to be broken down because when* I *was a pupil* I *don't feel that she* (the mentor's teacher) *broke it down enough, certainly not enough for me, and* so *whilst you can't say when you're in a top set at a grammar school that you struggled, there were people who could take far bigger steps than* 1. So *that was a very conscious thing, that when I started teaching, that it would be* so *small and that there would be the recall for children to feel happy before they went on. I did it when I first started teaching found it worked and carried it on since.*

> *J: Everything really all I need at the beginning of a lesson, really, is an idea of more of less what I am going to do.*

I suggest that if you were a student teacher you would get quite a different experience of, say, learning how to plan lessons, if you had A. or had J. as a mentor. Hence the rationale for teachers to have some sort of dimension of practice on which to position themselves.

Consider the following quotes in the light of the 'negotiation-precision'tension:

> P: So *you have to develop the skill of asking the right questions rather than giving the right answers.* So *it's extremely important,* I *think, that if you can get a student in on that really crucial experience, to be able to guide a pupil, say, she were taking a higher level, to try and get them, without telling them?? just pointing them towards ...*

This mentor is explaining how he wants to teach his student teacher how to adopt an extremely pupil-centered 'negotiated' position. The following mentor illustrates, with his example, how teachers do guide their pupils, via negotation, to a received, precise meaning:

> D: *In asking them a question like 'What is the chance of it snowing this Christmas?' and from experience, the response to those kinds of questions is '1 in 2' because all that is going through their mind is the fact that, well, it's going to snow or it's not going to snow, so it's '1 in 2'. So, then you start drawing out of them well, what does '1 in 2' mean? If I put it here on the probability line; its got an even chance. Then eventually they realise that it means that once in every two years its going to snow, then they think about 'has it?', and it hasn't. And they start, because they have got this image* (of the probability line), *they start adjusting where they can put the answer.*

In the following extract the teacher is positioned further towards the 'negotiated' end:

> *A: We've booklets for our children doing examinations at all levels. Now, we'd done them together, the children and myself, because there is no point in just teachers words coming out. It's a case of " We did it, are you happy with how we have put down? Well, we will change that, we'll leave that, yes it makes perfect sense to us now, we've done it together." When it got to the lowest level, and the children did it, it didn't have hardly any mathematical language in it, but as long as it was mathematically correct, and we know what we mean, and I know what they wanted to write down to make them happy with it , you know, like 'diagonal' meaning "from one corner point to another going through the middle", and if they understood that it was in their language, it wasn't perfect, but it was what they wanted and it was what they could use.*

None of the mentors I interviewed advocated a 'define your terms and prove your theorems' approach! Nevertheless we can conceptualise this approach in terms of the regulation precision tension.

I want to explore the tension between being mathematically 'creative' and 'mechanical' in terms of the pedagogy involved in teaching the attainment target of the National Curriculum known as "Using and Applying Mathematics". This attainment target is concerned with the processes, (rather than the content), of mathematics. Through its component strands of 'Applications', 'Communication' and 'Reasoning, Logic and Proof', the *"pupils will discover the real power of mathematics. They* (these strands) *are at the heart of mathematics"* (N.C.C. 1989, page D2); school pupils are, therefore, through this part of the curriculum given access to working *qua* mathematician. Hence the activities and tasks a teacher devises for pupils' learning in this area of the mathematics curriculum need to be "authentic" in the sense coined by Clayden *et al,* (1994), if the curriculum objective of pupils discovering 'the real power of mathematics' and working 'at the heart of mathematics' is to be realised. This 'Using and Applying' part of our curriculum can be the 'creative' counterpart to the 'mechanical' practising of algorithms which has been a dominant perception of what goes on in mathematics classrooms. And as this mentor's quote above indicates, perhaps requires mentors to act *qua* mathematician too:

> *Investigations demand more of the teacher than a set exercise.*
>
> Can you explain why?
>
> *Because there is a lot more finding out to be done, lot more time is involved, you need more time and you need to isolate yourself* o *think.*

There have been significant changes to the general c ltural climate in the UK over the past decade or so. The 1982 Cockcroft Report as a very important lever in making the 'mechanical - creative' tension explicit in the oft quoted paragraph 243. By listing various styles of teaching, teachers' aware ess was drawn to needing to provide a range of experiences in mathematics les ons: on one hand providing standard mathematical exercises after teacher exposit on, with the aim that children gain knowledge of a skill, and on the other hand, offe ing problem solving activities and the like, in which children's mathematical auto my would be given space to develop. Building on this 'received wisdom', there was over the next decade an institutionalising of 'investigations' in mathematics w ich were to be the vehicles for children's creativity in mathematics. However, here is concern that these 'investigations', conceived as creative, problem solvin activities do themselves turn into lifeless mechanical exercises (see Lerman 1989).

An example of a mentor struggling with this 'mechani aI-creative' tension in her own teaching:

> N: You see, but was I wrong to put that *(an inves igation)* in front of him without having done that myself, now that's t e question?
>
> *You answer it!*
>
> Well, I think I was, but then if I had have known the answer to that would I have guided him so that he didn't have the challenge of trying to work it out, because I think I would have said something ... you shouldn't leave it to chance, and yet I enjoyed working at the side of him, trying to get it, but I hadn't the time, you should prepare thoroughly.

Summary:

Each mathematics mentor is a teacher with her/his own way of teaching and the student teacher has both to relate to how s/he teaches and forge their own style under the mentor's super-vision. The purpose, then, of offering these tensions is to:

(i) help the mentor be aware of the dilemmas that s/he works within on a daily basis

(ii) recognise a preferred equilibrium position in her /his practice

(iii) explain her /his practice and preference to the student, hence giving the student good access to her craft knowledge

(iv) be comfortable if the student's preferred ways of working are different from hers/his.

To counter the potential 'apprenticeship' role of initial teacher education (Maynard and Furlong 1993), the mentor could do with being equipped with various frameworks which give her/him a perspective of her/his position and help develop her /his reflective practice both as a school teacher and a teacher educator. This is what I have offered above. It would be appropriate to consider other 'tensions'. For example, the tension between using results from mathematics education research about how children learn, or misconstrue, and one's own intuition about this.

Apart from the Mathematical Association's publication *Mentoring in Mathematics Teaching* (Jaworski and Watson (eds) 1994) there seems to have as yet been little subject specific work on mentoring. So the constructs discussed above are offered in the spirit of 'how do we talk about variations in practice in a way that fits with mentors' own pedagogical mathematical knowledge?' rather than prescriptions and are intended to persuade others to offer their construals.

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