CONVERSA TION AS A METAPHOR FOR MA THEMA TICS AND LEARNING

Paul Ernest

Much can be learned about theories in mathematics education and elsewhere by exarrumng their underlying metaphors. Elsewhere I have compared the underlying metaphors of mind of various learning theories (Ernest, 1993). For example, I have argued that the metaphor of mind for information processing constructivism is that of a computer, an unfeeling thinking machine; that of radical constructivism is an evolving and adapting but isolated organism, a cognitive alien in an unknown or hostile environment; and that of social constructivism is that of persons-in-conversation. It is this last metaphor that I wish to pursue (and extend) here.

For some years I have been developing a social constructivist theory of mathematics and learning (Ernest 1991a), and persons-in-conversation plays the part of the central metaphor in this theory in a number of ways. In this paper I wish to indicate how this metaphor can be developed to describe mathematics itself, mind and the teaching and learning of mathematics.

The term 'social constructivism' is used in a variety of ways by different people (and indeed, by myself, over time). My view of the problematique of social constructivism is twofold. It comprises, . first, an attempt to answer the question: How to account for the nature of mathematical knowledge as \ scrcially constructed? Second, How to give a social account of the individual's learning and construction 0f,. mathematics? This account must accommodate both the personal reconstruction of knowledge, and. personal contributions to accepted mathematical knowledge. Below I sketch the contribution the metaphor of conversation makes to both of these problem areas. However I should indicate that in the second area my views have developed and shifted significantly. In Ernest (1991a) I attempted to bring together views of socialisation through language acquisition (drawing on Wittgenstein and Vygotsky) and a neo-Piagetion or radical constructivist account of the individual's construction of meaning. Although I offered criticisms of the radical constructivist position (e.g. Ernest 1991b) it is only gradually that I have come to realise how incompatible the neo-Piagetian/radical constructivist position is with a view of mind thorougWy based on the metaphor of conversation. So here by social constructivism I mean *socia!* constructivism, and not social *constructivism.*

CON VERSA **TION: AN EMERGENT METAPHOR**

Conversation in the form of written dialogue has been used in philosophy from the time of the Ancient Greeks. It has also been explicitly adopted as a central epistemological concept by many philosophers and theorists. But I need to distinguish between three different meanings of the term. Conversation originates at the interpersonal level, where persons in one or more shared 'forms of life' engage in direct conversations, based in common 'language games' (Wittgenstein). This living, actual conversation with others in real-time is based on shared experiences, understandings, values, and mutual respect. At this level conversation is one of the basic modes of interpersonal human interaction., perhaps even the most basic one, if understood inclusively enough.

Mediated forms of conversation involving written texts (understood broadly to include all forms of notation, inscription and sign systems) represent an important extension of the notion (Gadamer). However, the transition from spoken to written textual forms of conversation is a crucial one. It creates a different relationship between the author and what is uttered, and allows the text to be objectified and preserved beyond the moment of utterance. It allows mathematical texts, proofs in particular, to be construed as monological, with all answers anticipated and incorporated in the text. However, the reading of any text remains dialogical, with readers interrogating the text and creating answers from it. Thus the second form of conversation is at the cultural level, the 'conversation of humankind' (Oakeshott). This is the direct sum of interpersonal conversations in oral cultures. However the rich, complex, symbolic culture of the history of mathematics as we know it is only possible through extended conversation, based on the production and use of texts in pennanent fonn (but not limited to just that).

Third, there is internalized private conversation. Many theorists including Plato, Gergen, Harre, Mead, Shotter, and Vygotsky argue that thought itself is internalized conversation, and that socially situated conversation between persons plays a crucial role in the fonnation of mind. Consequently it is also a central underlying feature of the subsequent use of mind. Even the private and individual functions of mind are socially constructed, although once fonned they can take on a life of their own and operate a long way removed from any collective or public conversation. Mathematicians for example, can operate in isolation for extended periods of time, having internalised some of the conversational roles and procedures they learnt through conversation of the first and second kinds. These include, most notably, the role of proponent, in which a line of thinking or thought experiment is followed through sympathetically, for understanding, and the role of critic, in which it is examined for weaknesses and flaws.

All three forms of conversation are social, be they interpersonal, cultural or intrapersonal, for they involve an alternation of human voices: present or removed, real or imagined. Conversation is also a construction, as are all linguistic and other cultural entities and phenomena. My proposal, therefore, is to adopt conversation (understood broadly to accommodate its three aspects) as a basic epistemological notion for a social constructivist philosophy and theory of mathematics.

The argument for accepting conversation as epistemologically basic is that language and discourse play an essential role in the genesis, acquisition, communication, fonnulation and justification of all knowledge, mathematical knowledge in particular. Conversation is the dialogical deployment of language, with its social exchange dimension, and its dialectics, with ebb and flow, assertion and counter assertion, is essential for communication and feedback, in the fonn of acceptance, elaboration, reaction, criticism and correction. This underpins the justification of objective mathematical knowledge, and the ratification of personal knowledge, as I describe below.

In general epistemological terms, Collingwood, for example, similarly proposes a dialectical 'logic of question and answer' in place of the (mono) logic of propositions. Again, Rorty adopts conversation explicitly as his philosophical basis for epistemology and mathematical knowledge.

> If, ho\Vever, we think of "rational certainty" as a matter of victory in argument rather than of relation to an object known, we shall look toward our interlocutors rather than to our faculties for the explanation of the phenomenon. If we think of our certainty about the Pythagorean Theorem as our confidence, based on experience with arguments on such matters, that nobody will find an objection to the premises from which we infer it, then we shall not seek to explain it by the relation of reason to triangularity. Our certainty will be a matter of conversation between persons, rather than an interaction with nonhuman reality.
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> Rorty (1979: 156-157)

CONVERSA TJON IN MA THEMA TICS

Currently, there is a move in some quarters to reconceptualise mathematics and the philosophy of mathematics in fallibilist, human-centred and even social tenns (Davis and Hersh, Kitcher, Lakatos, Tymoczko, Ernest 199Ia).

This reconceptualisation represents a break from the traditional absolutist views of mathematical knowledge which see it as monological in character. Monologicality is a central assumption of Cartesian rationalism and the modernist outlook based on it. Mathematical knowledge is presented as if it is God-given, not uttered by human voice, let alone by a one of several voices (albeit a dominant one) in a dialogue or conversation. Monologic thus produces perfectly ordered and structured texts with no trace of author/listener, with an implied single meaning, which is the utterance of Authority. It can be viewed as either a perfect polished achievement of dialogue; or a degenerate fonn employed by Authority to impose power (in mathematics, society or the classroom).

Instead, my argument is that mathematics is dialogical, and that conversation permeates mathematics in deep and multiple ways. The underpinning metaphor of conversation stresses dialogic, comprising alternating voices in a shared quest for understanding, based on the logic of question and answer, and on uncertainty. This resonates with the fallibilist turn in the philosophy of mathematics. However, the claim that mathematics is conversational, dialogical or dialectical can be understood in multiple ways (I do not distinguish these three notions here, but use them as loosely equivalent.). These include: its linguistic/textual basis; its concepts and content; the foundations of proof; and the underlying epistemology and methodology of mathematics.

Mathematical Language

Mathematical activity is primarily a symbolic activity, which uses written inscription and language to create, record and justify its knowledge (Rotman, 1993). Viewed semiotically as comprising texts, mathematics is inescapably conversational and dialogical, for by its very nature it addresses a reader (V olosinov, Bakhtin). Beyond this general feature, an analysis of mathematical texts, proofs and algorithms, reveals the verb forms employed to be both in indicative and imperative moods. The declarative case of the indicative mood is used by the writer to make statements, claims and assertions, which are claims about the outcomes of certain processes. The imperative mood is used for both inclusive and direct imperatives, which are shared injunctions, or orders and instructions issued by the writer to the reader. Thus mathematical texts comprise specific assertions and imperatives directed by the writer to the reader, i.e. they are one-sided segments of dialogue (Rotman, 1993).

Mathematical proof is a special form of text, which since the time of the ancient Greeks, has been presented in monological form. This reflects the absolutist ideal that total precision, rigour and perfection are attainable in mathematics. Thus the monologicality of the concealed voice uttering a proof itself belies and denies the presence of the silent listener. But as it is an argument intended to convince, a listener is presupposed. The monologicality of proof tries to forestall the listener by anticipating all of her possible objections. So the dialectical response is condensed into the ideal perfection of a monologic argument, in which no sign of speaker or listener remain.

Mathematical Concepts and Content

A substantial class of modern mathematical concepts and content have an underlying conversational or dialectical basis. These include, for example, aspects of analysis (e-a limit definitions: "You give me e, and I'll give you a"), statistics (hypothesis testing: Ho versus H_A), probability (analysis of wagers, betting games), game theory (alternation of moves by opponents), constructivist logic (the interpretation of quantifiers 'v'x3y: "You choose *x*, and I show how to construct *y")*, number theory (John Conway's game theoretic foundations of number), set theory (game theoretic version of Axiom of Choice, Cantor's dialogical diagonal arguments), recursion theory (interpretation of quantifiers in arithmetical hierarchy). Thus it can be said that conversational and dialectical interpretations can be given to a significant range of concepts from some of the main branches of mathematics, and form an a necessary characteristic of some others. Thus the dialogicality of mathematical content is widespread and deep.

Origins and Basis of Proof

Dialectics and conversation provides the origins of mathematical proof and logic, and a foundation for certain modern conceptions of logic and proof. Mathematical proof, certainly in its axiomatic form, developed in Classical Greece, probably due to the widespread practices of disputation and dialectical reasoning, which were central to the public democratic institutions and cultural practices of the day. (The word 'dialectic' is derived from the verb meaning 'to discuss'.). Szabo and others locate the source of deductive mathematics and logic in dialectical argument, disputation and conversation. Thus it seems that Rorty's conversational reading of persuasion in proof reflects its very origins, and is not just a postmodem reading of it.

In proof theory, some of the main developments also treat mathematical proofs as if they are offered in a dialogue. In them a proponent attempts to convince an opponent of her claims, whilst the opponent challenges what is asserted, but accepts a number of agreed basic rules of reasoning and facts). Thus these developments are evidently dialogical. They can be found in Heyting's intuitionistic proof theory, Natural Deduction, the method of Semantic Tableaux, and in Lorenzen's constructive logic (used by Habermas as a basis for his conversational Theory of Communicative Action). Hintikka also proposes a system of Game Theoretic Semantics for tableaux.

Thus both the beginnings of logic and mathematical proof and many of their modem developments suggest that mathematical proof is at root dialectical, based in human dialogue and on conversational exchange.

Epistemology and Methodology

The epistemology and methodology of mathematics, including the nature and mechanisms of mathematical knowledge genesis and warranting can be accounted for in an explicitly and constitutively dialectical way. The social constructivist account of the conversational basis of mathematics is based on primarily on the work of Wittgenstein and Lakatos (Ernest 1991 a, forthcoming). Wittgenstein offers the basis of a social theory of meaning, knowledge and mathematics resting on dialogical 'language games' embedded in 'forms of life'. This basis makes it clear that conversation rests on shared experiences, habits, understandings, assumptions and participation in communal activities. Wittgenstein thus shows the situated, contextual basis of all knowledge. He also provides an account of mathematical proof and necessity, based on socially accepted notions of 'following a rule' as opposed to the objectivity of mathematical knowledge understood in an absolutist or transcendent way.

Lakatos offers a multifacted if incompletely formulated theory which crucially reintroduces history into the philosophy of mathematics. At the heart of this is his heuristic or Logic of Mathematical Discovery (LMD), which is a dialectical theory of the history, methodology and philosophy of mathematics. Lakatos' LMD can be explicated as a cyclic process in which a conjecture and an informal proof are put forward (in the context of a problem and an assumed informal theory). In reply, an informal refutation of the conjecture or proof are given. Given work, this leads to an improved conjecture or proof, with a possible change of the assumed problem and informal theory. This pattern is evidently conversational and dialectical.

> The proof procedure seems to me to be a remarkable example of the dialectic triad of thesis, antithesis and synthesis. The progress of mathematical thought - in this case starts with the primitive conjecture. This is the thesis. This thesis produces its antithesis which consists of the tension and struggle of the proof and refutations .... Now the synthesis is the theorem which embodies the respective values of both poles of the antithesis - proof and refutations - on a higher level, without the limitations of both.
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> Lakatos (1961: 51)

To overcome some of the criticisms directed at Lakatos, this scheme can be generalised to accommodate a broader range of changes, outcomes and responses including theory growth and 'mathematical revolutions' (Ernest forthcoming). Following this scheme, mathematical proofs or other proposals are offered to the appropriate mathematical community as part of a continuing dialogue. They are addressed to an audience, and they are tendered in the expectation of reply, be it acceptance or critique. Such replies may play a part in the development and formulation of new mathematical knowledge. However, such replies, when given by the gatekeepers of institutionalised mathematical knowledge (e.g. journal editors) play the essential warranting role in the acceptance (or rejection) of candidates for new mathematical knowledge. The social acceptance of mathematical knowledge (and hence its status as knowledge) is constituted by this conversation.

Within the contexts of professional research mathematics, individuals use their personal knowledge both to construct mathematical knowledge claims (possibly jointly with others), and to participate in the dialectical process of criticism and warranting of others' mathematical knowledge claims. In each case, the individual mathematician's symbolic productions are (or are part of) one of the voices in the warranting conversation.

Thus mathematical proof has not only evolved from a dialogical form, but its very function in the mathematical community as an epistemological warrant for items of mathematical knowledge requires the employment of that form. The underlying logic is dialectical.

CONVERSA TION IN LEARNING MA THEMA TICS AND MIND

conversation and mind

A number of theorists, including Mead and Vygotsky, argue that thought itself is internalized conversation. They claim that thought is constituted and formed by intrapersonal conversation, and that thinking: is internalised conversation with an imagined other. On this basis, mind can be viewed as social and conversational because first of all, individual thinking of any complexity originates with and is formed by internalised conversation; second, all subsequent individual thinking is structured and natured by this origin; and third, some mental functioning is collective (e.g. group problem solving).

Harre has developed a theory of Vygotskian space based on two polarities of thinking or speech, comprising: manifestation or display (public or private) and social location (collective or individual). He combines these in a Cartesian product to make four quadrants which have a cyclic relationship in the development (and location of mind), and which also defines the cycle of the appropriation and testing of knowledge. Harre uses the terms 'appropriation', 'transformation', 'publication', 'conventionalisation' to describe the successive passage of thought and knowledge (and even the construction of personal identity) from one quadrant to the next. This closely resembles the social constructivist theory of the cyclic passage of mathematical knowledge from 'objective' (i.e. public) to subjective and then back to objective knowledge again, shown in Ernest (1991 a: 85, forthcoming). It also fits with theories of socialisation.

> society is understood in terms of an ongoing dialectical process composed of the three moments of externalization, objectivation and internalization ... .In the life of every individual, therefore, there *is* a temporal sequence, in the course of which he is inducted into the societal dialectic. The beginning point of this process is internalization ...
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> Berger and Luckmann (1966: 149)

Thus the dialectical process of the formation of mind and of individuality is understood to be dialogical and conversational. This is the model of mind-world relations adopted by social constructivism in the current version (Ernest forthcoming).

Conversation in the Teaching and Learning of Mathematics

Mathematics, like any other area of knowledge, is learned through individuals (learners) participating in language games embedded in forms of life. Personal knowledge or competence in mathematics is acquired through prolonged participation in many socially situated conversations in different contexts with different persons. Initially, the forms of life are domestic and out of school, and these provide an essential set of capabilities for young persons to enter into the novel, formalised learning settings in schools and other educational institutions. Schools, of course, only represent one cluster of contexts and social practices into which young learners enter into and learn from. These are planned teaching and learning situations in which the teaching of mathematics is deliberate. In the context of such intentional forms of mathematics education (in or out of formal institutional settings) certain individuals (teachers) structure mathematical conversations on the basis of their own knowledge, and texts, in order to offer mathematical experiences to learners, with the aim of developing their mathematical competences. They direct, structure and control mathematics learning conversation both to present mathematical knowledge to learners directly or indirectly (i.e. teaching), and to participate in the dialectical process of criticism and warranting of others' mathematical knowledge claims (i.e.

assessment). These two functions are irrevocably intertwined, except in their extreme forms where they are temporarily and conventionally separated (e.g. expository lecturing and marking external assessments) .

The learning conversation extends beyond the immediate teacher-pupil interaction. In school contexts, there are attenuated conversations including leamer-textually presented answer interactions, leamer-computer presented answer interactions, learner-peer interactions. In out-of-school contexts there are in addition to the above, leamer-parent and leamer-significant other interactions.

The public representation of mathematical knowledge within a teaching-learning conversation (including its textual variants) is necessary but not sufficient for such knowledge to become the personally appropriated mathematical knowledge of an individual learner. Sustained two-way participation in such conversations is also necessary to generate, test, correct and validate mathematical performances. Teacher-pupil dialogue (usually asymmetric in classroom forms) typically takes place at two levels: spoken and written. In written 'dialogue' pupils submit texts (written work on set tasks) to the teacher, who responds in a styli sed way to its content and form (ticks and crosses, marks awarded represented as fractions, crossings out, brief written comments, etc.). The primary aim of such conversation is that of ensuring that the learner is appropriating collective mathematical knowledge and competences, and not some partial or distorted version. Appropriated mathematical knowledge is potentially unique and idiosyncratic, because of human creativity in sense-making. This possibility also arises because school mathematical knowledge is not something that emerges out of the shared meaning and purpose of a pre-given form of life. Instead it is a set of artificially contrived symbolic practices whose meaning is not already given, but is deferred until the future, or at least a significant part of it is.

What must not be overlooked is that conversation is fundamentally a moral form, not just about exchanging information. For it entails engaging with a speaker or listener as another human being, not just as a source or end-user of information. Thus in education the use of the conversational metaphor in the teaching and learning of mathematics ideally should entails a number of things. For a start:

- Mutual respect and trust between teacher and learner;
- Listening to learners; showing (and feeling) an interest in their views, in their conceptions, and in their sense-making;
- Making teaching into *real* conversation, into a *real* dialogue where there is respect for the leamer's intelligence and where there is space for learner initiative too;
- Treating real subjects and content of mutual interest and of mutual benefit.

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Author's address for correspondence

Dr. Paul Ernest, University of Exeter, School of Education, Heavitree Road, Exeter, Devon EX} 2LU, United Kingdom. Tel. 0392-264857, Fax 0392-264736, E-mail: PErnest@cen.ex.ac.uk