Students' Understanding of Differentiation and Integration

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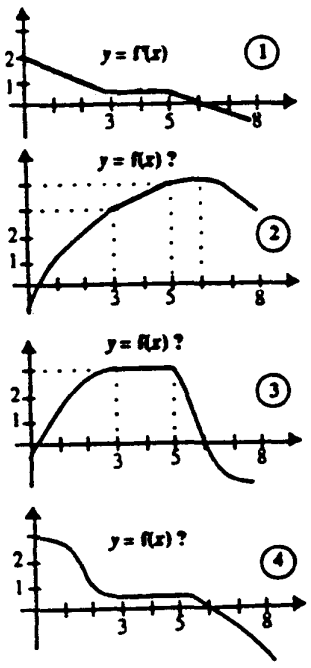
This study is a small part of a wider undertaking which examines students' cognitive difficulties in understanding differentiation and integration in both computer and non-computer environments. Here the results of a pilot study which took place at sixth form college are described and discussed.

The focus here is to analyse A-level students' errors and difficulties on a question. This question (taken from Cornu 1983) shows a graph which is the derivative of one of three others. The students have to choose one, and give reason(s) for their opinion. This question (see Figure) were administered to 18 students who have been using graphic calculators through the course.

Graph 1 is the derivative y = f'(x) of a function y = f(x) defined for $0 \leq x \leq 8$.

Which of the graphs 2, 3, 4 could be the original graph y = f(x)?

Give reason(s) for your choice.



Two of those students could not attempt the question. Among those students (9 students) who gave correct answer only one of them made a full explanation, the others gave partial explanation:

" Between 1 & 6 f'(x) is positive so f(x) is increasing, and after x = 6, f'(x) is negative, so f(x) is falling. Also on (2) the line becomes straight as f'(x) crosses the x-axis."

" after x = 6, there is a negative gradient."

Seven of the students chose the wrong answer in this item. Three of those students chose graph 3 although four of them chose graph 4. A typical explanation given by a student who chose graph 3 is given below:

"It shows the stationary points in the correct areas."

Even more interesting are the explanations of those who chose graph 4:

"The gradient graph does not increase at any point."

" Crosses x- axis at 6. Level between stages x = 3 and x = 5."

Briefly, the results show that 50% of the students gave the correct response with a correct reason. Empirical research done by Tall (1986a) also revealed that 67% of the experimental students who used Graphic Calculus (Tall, 1986b), chose the right answer with a correct explanation, while only 8% of the control students did. Thus it is likely that visualization in the graphical context can help students to understand the relations between differentiation and integration.

In the wider study the following will be addressed.

•What are the misconceptions beyond these difficulties?
• Is it possible to diminish or eliminate these difficulties with the use of computer?If so, How?

References

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