

## **Introduction**

The aims of this project were to investigate the metacognitive skills and concepts possessed by students of secondary school age in some typical mathematical learning environments, to explore the feasibility of raising the levels of their awareness by appropriate interventions, and to study the effects of such enhancement on the students' mathematical attainments.

The outcomes of the project include, as well as these results detailed below; the Teachers' Handbook containing the set of suggested enhancement activities, trialled and including examples of students' work; the Evaluative Instruments, partially developed but needing further improvement; and the set of Case Studies of the seven classes during the main experimental year. (*Documents available at present from the Shell Centre are the Summary Report (£1), the Teachers' Handbook (£6), Evaluation (£6)*)

Our interest in students' awareness of their learning arose from our work in a previous ESRC project *Diagnostic Teaching in Mathematics* (ESRC 8491/1) (Bell et al, 1985). In this, a teaching methodology based on identification of students' concepts and misconceptions and resolution of the latter by exposure, cognitive conflict and discussion, proved to be strikingly more effective than more usual methods, particularly for longer term retention. What became clear was that such teaching methods demanded a radical change in the students' conceptions of what was appropriate activity in a mathematics lesson. An orientation towards obtaining correct answers had to give place to a recognition that the aim was to acquire correct, well knit concepts and methods, and that this involved being willing to expose ones own ideas and approaches, even if wrong, and to look for personal satisfaction in the enlightenment provided by participation in a focused discussion. This in turn depended on an awareness of the nature of this type of learning and its distinction from memorisation and fluency practice. This led us to consider the possibility of achieving improved learning across the whole mathematics curriculum by increasing students' awareness of learning methods and their purposes.

A substantial amount of experimentation in the encouragement of metacognitive activity in school and teacher education settings has been built around the PEEL project, based in Melbourne, Victoria, Australia (Baird and Mitchell, 1986; Baird and Northfield, 1992). In this project, a substantial number of teachers at a particular school worked concertedly at developing methods by which the students (aged 15-16) took greater control of their own learning.

An experiment somewhat similar to our own, but with a single class of primary school (year 6) children, was conducted by Herrington (1992). His one-year programme sought to improve learning strategy awareness, mathematical achievement and confidence towards learning mathematics; it used some 70 short interventions involving concept mapping, a Think Board, self-questioning and writing. Significantly better gains than those of a control group were shown on learning strategy awareness, and non-significant improvements in confidence and mathematical attainment.

Our own project has focused on enhancing reflective activities and on providing lesson experiences through which students may acquire specific knowledge about learning tasks and processes; and this in real classroom settings.

Aspects of awareness: Aims and objectives for the teaching and testing

*1. To increase awareness of the components of mathematical activity.*

Distinguishing facts and skills, conceptual structures, general strategies and appreciation and attitudes towards mathematics.

*2. To increase awareness of mathematical content.*

Knowing one's way round mathematics as a discipline, and being able to identify one's own existing state of knowledge.

*3. To increase awareness of mathematical strategies*

Knowing that acquiring mathematical strategies is such as these for problem solving and investigating a legitimate goal for learning; being able to distinguish, identify and thus ultimately to deploy them.

*4. To increase awareness of types and purposes of mathematical tasks*

Distinguishing, for example, the relative purposes of investigative practice tasks.

*5. To increase awareness of the purposes of different ways of working*

Distinguishing between activity and learning; appreciating that learning involves a positive orientation towards recognising what is available for learning, and seeking to *understand* as to become *more fluent* in it.

*6. To increase awareness of resources for learning and how to use them*

Selecting appropriate resources, including the teacher, other pupils computer or printed resources.

*7. To increase awareness of general learning principles*

Appreciating, for example, the value of previews/reviews or of discussing causes of errors.

## Evaluative Material

Three tests were produced covering different aspects of awareness .

. . The first 'evaluates awareness of the *purposes of different ways of working* and awareness of resources for learning, as well as exploring appreciation and attitudes towards mathematics. The second focuses on awareness of the *components of mathematical activity*, mathematical content and general strategies. The final booklet assesses awareness for *types and purposes of mathematical tasks*.

*Some Pretest Results from the 'Purpose~' (c):'*

Jane's maths teacher gave her questions like these questions to do.  
Jane did not use a calculator.

1)  $5.53 \times 6$   
2)  $23.4 \times 5$   
3)  $3.52 \times 2.5$   
4)  $3.25 \times 4.54$

This is what Jane did.  
She worked on her own.

The image shows Jane's handwritten work for the four problems. Problem 1:  $5.53 \times 6 = 33.18$ . Problem 2:  $23.4 \times 5 = 117.0$ . Problem 3:  $3.52 \times 2.5 = 8.800$ . Problem 4:  $3.25 \times 4.5 = 14.625$ . Each calculation is done vertically and has a checkmark next to it.

What was the purpose?

2 - a main purpose, 1 - helped a bit, 0 - not a purpose, ? - not sure

This activity is to help Jane ...

- (a) to practice multiplying quickly and accurately
- (b) to know *when* multiplication is the right thing to do
- (c) to learn to work neatly
- (d) to think about what decimal numbers mean
- (e) to remember the roles for multiplying
- (f) to get better at discussion and explaining (g)
- to see how to use maths in her everyday life

Percents  
choosing this as  
a main purpose

Yr 7	Yr 9	Yr 10
52	58	79
17	22	32
9	12	15
25	37	46
26	41	62
6	8	9
43	21	15

The main perceptions of the purpose of the task become clearer from year 7 to year 10; these are the kinds of increase that one might expect. Similarly, the confusion with everyday life shows a decline, again corresponding with expectation. Perhaps the most significant observations relate to the purposes (b) and particularly (d) which we might interpret as showing that the students do not discriminate at all well between tasks aimed at the development of manipulative skill and those aimed at comprehension.

## Ways of enhancing the meta cognitive aspects of learning

18 intervention strategies were developed, and appear in the Teachers' Handbook

### *Reflection and review activities*

- a Students making up questions
- b Students reflecting on learning difficulties and misconceptions
- c Students reviewing and classifying  
(Including students interviewing students, drawing concept maps  
compiling review booklets, and planning an outline for a mathematics textbook)

### *Students as assessors*

- d Students constructing tests
- e Students devising and using marking schemes
- f Students diagnosing errors
- g Students assessing themselves against statements of attainment
- h Students predicting their own performance

### *Students as teachers*

- i Students teaching students

### *Students learning from text*

- j Students writing meanings for headings and subheadings
- k Students using terminology and definitions
- l Students surveying the structure of text
- m Students sequencing pieces of text
- n Students composing text

### *Students reflecting on ways of working*

- o Students conducting mini-debates
- p Students conducting small group discussions
- q Students observing students
- r Students describing what learning feels like

Many of these incorporate student role-reversal activities, for example, student as a teacher, an assessor, a textbook designer or an observer.

The effectiveness of the interventions

### *Reflection and review activities*

These activities were readily seen by students as purposeful; student-produced posters and booklets designed to introduce newly arriving students to mathematics were one manifestation of this. *Diaries* on the

other hand, did not work well; they soon became a routine chore (as found also by Baird and Mitchell (1986).

#### *Students as assessors*

These interventions were used frequently and enabled students to appreciate more fully the purposes underpinning activities. Before students assessed work they were often called upon to devise a marking scheme; consequently they had to identify and order the underlying task objectives. This type of role reversal activity also provided a means through which errors or misconceptions could be constructively addressed.

When accompanied by a discussion of the relationships between topics and their relative importance *students constructing tests* provided a means of enhancing awareness of content. This technique was a useful diagnostic tool because it enabled teachers to determine the match between their intentions and the learning outcomes, and for students it provided a means through which they could realise more fully their own level of understanding, and on occasion, lead to a re-evaluation of their own state of knowledge. *Students constructing questions* was not confined to end of unit test construction; this ongoing intervention was also frequently used to enhance awareness of content. (See the illustration below)

#### *Students as teachers*

This was used by only two teachers, because it involves considerable time and relatively complex organisation and collaboration. For it to work successfully it seems essential that student-teachers are given time to prepare their methods and materials. On one occasion where adequate time was provided, the student-teachers were well motivated and organised.

#### *Students reflecting on ways of working*

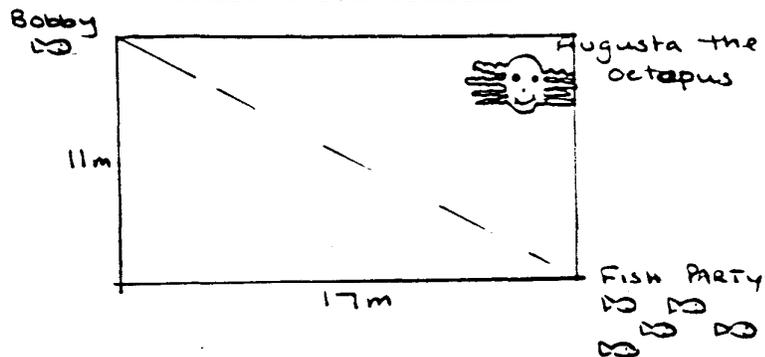
The review activity, *students interviewing students* was not trialled extensively but the available evidence suggests that it has value and is worthy of further investigation.

*Students as observers* was less frequently used, but the implementations that did take place provided positive outcomes.

*Students conducting mini-debates* was used by seven teachers, who all rated it highly. It involved students in constructing and defending arguments for and against different ways of working and resulted in vigorous and reflective discussion. It was adopted for in-service training. Some of the cards giving the students' briefs are shown below.

Alan Bell

## BOBBY'S BIG PROBLEM



Bobby the fish is going to a party, but as he is swimming along he spots Augusta the octopus. Bobby has said he will babysit for Augusta's baby octopuses when she goes out, and she looks as if she is about to go out, but Bobby doesn't want to babysit now because he is going to a party.

He can swim 21 metres before Augusta can corner him. Will he reach the party?

## Debating Cards for Group A

Cut out these cards and share them among your group.

Between you, prepare a 2 minute argument supporting each statement.

Share out this work so that it does not take too long.

**1. You learn more from working on one hard problem, than from working on ten easy problems.**

**2. You learn more from getting things wrong than from getting things right.**

**4. You learn more by trying to explain something yourself, than by listening to someone else explaining.**

**5. You learn more by working on a lot of short problems than by working on a few longer investigations.**

**7. You learn more from listening to a good explanation than by working on problems .**

**8. It doesn't matter if you copy someone else's work, so long as you understand it.**