Using prompt videos to improve problem-solving skills

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The article describes the development of a resource to support candidates for the United Kingdom Mathematics Trust's (UKMT) Challenges. It describes how the Challenges were introduced in a school and how they work. The data provided by the UKMT in its Yearbooks and the pedagogical value of the Challenges as a means of encouraging problem-solving are considered. The interventions undertaken to support candidates over the years, which culminated in the development of the prompt videos, are discussed. Academic research on prerequisites for teaching and learning problem-solving is addressed and leads to a description of how prompt videos address students' difficulties. The program of research and work for the next year is outlined.

Keywords: online resource; secondary; problem-solving

Introduction

This Action Research has taken place over the last two decades in an Asian-British independent school which has now closed. The school selected from the top half of the ability range and performed well in Mathematics. The teaching of Mathematics was transmissional and the students were well trained in book work in the primary department. The only exception to this style of teaching was the investigative and statistical GCSE coursework before it was removed from the specification. The style of teaching and learning was accepted by students, teachers and management. The parents of the students also expected it. In such circumstances, well delivered transmissional teaching led to the good examination results which are generally taken as a sign of a good Department. At least, complaints were rare.

The Challenges of the United Kingdom Mathematics Trust (UKMT) were introduced in 2004 simply because it was believed that the students would enjoy them. Problem-solving type questions are often to be found at the end of text-book exercises which begin with more routine bookwork. Such questions are often neglected. Dedicating a fixed period of time annually to this important, or even essential, part of the subject proved to be popular and the three basic Challenges became a settled component of the provision of Mathematics at the school.

The Challenges

The three main UKMT Challenges have a multiple-choice format and consists of 25 questions. The questions have to be answered within one hour for the Junior Maths Challenge (JMC) for the Years 7 and 8 (12 & 13 year-olds), and for the Intermediate Challenge (IMC) for the Years 9,10 and 11 (14,15 & 16 year-olds). The Senior Mathematics Challenge (SMC) for years 12 and 13 (17 & 18 year-olds) has the same format but the students are allowed 90 minutes. Some 40% of candidates are awarded either Bronze, Silver or Gold Certificates and the highest scorers may be invited to

participate in further competitions such as the Kangaroos and various Olympiads. A further process of selection yields UK teams for the International Olympiads. The Challenges and further competitions therefore provide well-crafted and stimulating problems for students of many levels of competence.

Participation in the Challenges raised the profile of the subject in the school. The Certificates were distributed in school assemblies and at the annual Prizegiving. As the Challenges occur in Winter (SMC), Spring (IMC) and Summer (JMC) terms, the Mathematics Department and the award winners appeared regularly before the school. At the minimum, it maintained its presence and its significant profile among the other subjects.

A small number of students were invited to participate in the Kangaroo Challenges and a very few entered in the Olympiads which are designed to stretch the very able. The number of awards increased and a pilot study suggested that the mean number of questions answered correctly did rise.

Over the years, a number of students developed a real passion for the subject and chose to study the subject at degree level. It would be difficult, and perhaps not even possible, to quantify the effect of using the Challenges within a school. To all appearances, within this particular school, participation enhanced the enjoyment of the subject and helped to open up some students' view of Mathematics.

The United Kingdom Mathematics Trust

The UKMT is a highly successful charity founded in 1996. Its charitable purpose is "to advance the education of children and young people in Mathematics." (UKMT Yearbook 2018-19, p.2) Participation in national mathematics competitions was running at about 24,000 in 1988 and grew vigorously by one order of magnitude until 1995. Since the various competitions were brought together by the founding of the UKMT, numbers have risen from 250,000 to 700,000 and continue to grow.

Its Challenges have been well described as the most reliable source of enrichment for secondary mathematics in the UK. As can be seen from a perusal of their website https://www.ukmt.org.uk a team of highly skilled problem-posers, reviewers and editors are at work behind the scenes as well as the administrators and staff.

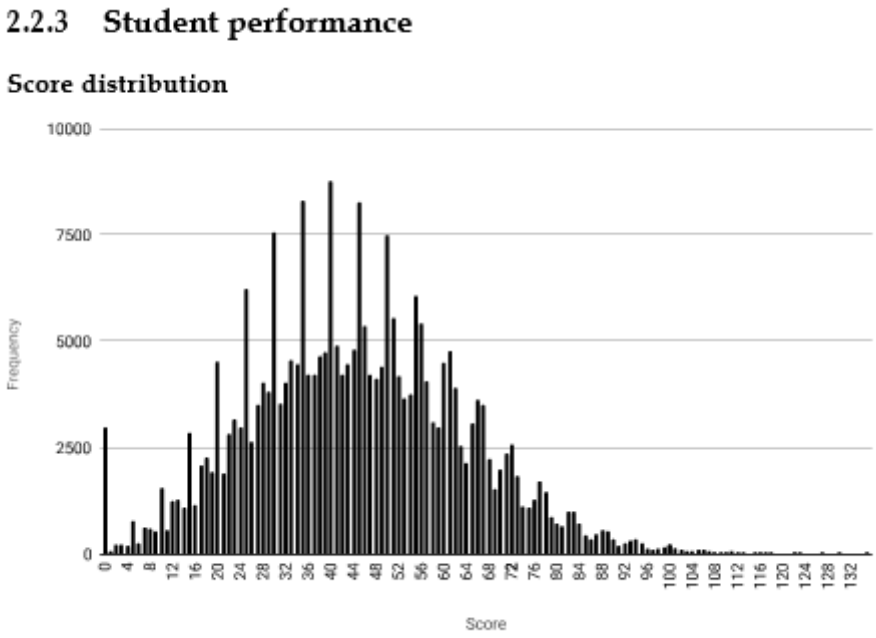
In 2019, when the then Deputy Director was asked whether the UKMT had ever been subject to any attention from the community of researchers in Mathematics Education, he said that he was unaware of any. The UKMT has grown quietly and organically to become an important component of mathematics education in the UK. It may be argued that as a significant phenomenon it is currently under-researched.

Observations on the data provided in the UKMT Yearbooks

The Yearbooks provide a meticulous record of the activities of the Trust during each year. It also provides the percentages of total responses for the five possible answers and a simple commentary on those percentages. Given the numbers taking the Challenges, it may be possible to derive up-to-date information on the particular difficulties students across the country are encountering and compare it with the difficulties faced in previous years.

The distributions of scores in the Year's Challenges are also presented in graphs (figure 1).

Figure 1: The score distribution given for the JMC in 2018 (UKMT Yearbook 2017-2018, p.13)



Without conducting tests, one can see than the distribution is fairly normal with a slight positive skew. The spikes are accounted for by the scoring system. Correct answers to questions 1 – 15 are awarded 5 marks each. 6 marks are awarded to correct answers to questions 16 – 25 but marks are subtracted for wrong responses. Score totals which are multiples of 5 occur more frequently.

There are significant questions about whether the best way to help young people enjoy mathematical problem-solving is to give them one hour to solve 25 problems for which the method of solution is not known in advance. In this JMC we see that some 3000 candidates scored no marks at all which is a little more than 1% of the 276,638 who returned scripts. Estimating from the diagram, some 15% answered 5 questions or fewer correctly.

The school in which this research was conducted selected students from the top half of the ability range and scores of zero were almost never seen. Nevertheless, there was a negative affect associated with the Challenges for those who achieved little in the allotted time. In the conversation already mentioned, the previous Deputy Director of the UKMT remarked upon one school which always entered students but never achieved any awards. Such a situation might be one in which a user-friendly online resource might be of value.

These points need to be set in the context of the general difficulties of teaching and learning Mathematics, in which context it has to be said that failure is not uncommon. Again, on the other side of the mean are those who succeed in answering more than roughly eight questions and those who achieve awards of Bronze, Silver and Gold Certificates etc. The awards give clear confirmation of the candidates' ability in Mathematics and they enjoy valuable moments of encouragement and praise.

Interventions and the development of the online-resource

As the years passed, the question arose how best to support preparation for the Challenges. It might be said that the action research began with that question. From 2008, the primary department was encouraged to enter students for the Primary Mathematics Challenge (PMC). The PMC is a distinct organisation which is not under the umbrella of the UKMT.

From 2009 they were encouraged to enter selected students from years 5 and 6 for the JMC. That proved to be a success. Packs of past papers were provided to all candidates for all the Challenges to practice at home. Some parents and siblings enjoyed doing the problems with their children. Lesson time before the Challenges was allocated to problem-solving across the school.

During these years technology was improving and new possibilities emerged for supporting the students. There was a constraint on the time which could be made available in the timetable which arose from the powerful focus on good examination results. But videos now could be made which read the question to the student and provided prompts to engage the viewer and gave them the opportunity to complete the question in their own time. These videos could be watched at home by those students with the will to prepare for the Challenges, as many turned out to have.

These videos were first housed on the server of the Mathematics Department's computer room. Students were provided with earphones, the relevant past papers and access to the videos made for each of the twenty-five questions. On occasion they became so deeply absorbed for forty minutes that the supervising teachers had an opportunity for sustained quiet reflection.

In the next stage, the videos were posted on YouTube with restricted access and the links were attached to a Googlesheet which was made available to our students so that they could view the videos at home. In 2019 the UKMT became aware of this work and kindly gave copyright permission for these prompt videos to be published on YouTube provided that no commercial gain was involved. It was possible to offer the videos to other schools and they enjoyed a certain amount of usage.

In the most recent stage, it has been possible to build our own website, https://www.challengeprompts.co.uk to house the videos and make the interface as friendly to the user as possible. On the website, the viewer is first presented with the text of the question and five buttons to click on for the five possible multiple- choice answers. If the response is correct, the word 'correct' appears, and, if not, the viewer is presented with the relevant prompt video. The website can be used as a general problem-solving resource and by individuals or groups.

Academic research on teaching and learning problem-solving proficiency

Problem-solving has always been regarded as a distinct skill as compared, for instance, with arithmetical fluency. It is often ranked as the most important skill. "The mathematician's main reason for existence is to solve problems … therefore, what mathematics really consists of is problems and solutions" (Halmos, 1980, p.519).

Chapman (2015) considered the skills which are needed to teach mathematical problem-solving effectively. In order to establish a list of distinct components of both problem solving and mathematical proficiency, a literature review of articles dealing with problem-solving in relevant journals dating back a century was conducted. The

distinguishable components of these proficiencies naturally overlap and are seen to be interdependent.

There is a notable convergence in the most recent descriptions of the components of mathematical and problem-solving proficiency which is presented in the slightly adapted Table 1 below.

Table 1: Perspectives of Mathematical Problem-Solving Proficiency

Mayer and Whittrock 2006	Schoenfeld 1985	Kilpatrick et al., 2001	Mathematical Problem-Solving Proficiency	
Concepts	Appropriate Resources	Conceptual Understanding	Understanding of mathematical concepts, operations and procedures	a
Procedures		Procedural Fluency		b
Strategies	Heuristic Strategies	Strategic Competence	Understanding of general heuristics and specific strategies and when and how to use them.	c
Metacognitive Knowledge	Metacognitive Control	Adaptive Reasoning	Capacity for logical thought and reflection and the awareness, monitoring and controlling of one's own cognitive processes during problem-solving	d
Beliefs	Appropriate Beliefs	Productive Disposition	Holding beliefs about mathematics and one's problem-solving capacity that support motivation and competence	e

In the article, it is observed that teachers of problem-solving will require "knowledge of students as problem solvers" (Chapman 2015 p.26). Early studies of students as problem-solvers between 1922-44 focused on the difficulties they had which made teaching problem-solving "one of the hardest and most discouraging tasks of the teacher" (Washburne & Osborne, 1926, p.222). The vocabulary of the list provided is negative in tone – inability to read, inadequate understanding, carelessness, failure to comprehend, ignorance, insufficient mastery, lack of interest etc. – and seems to suggest that the students were wholly responsible for their difficulties.

Nonetheless the points raised may be readily mapped to the five perspectives in Table 1 with two differences. The concept of metacognition – or presumably the absence of it – does not occur in the early list of difficulties. The important question of the ability to read does not occur either in the table of perspectives or in the more positive list of characteristics of good problem-solvers derived from post-war articles (Chapman, 2015, p.28) Any practicing teacher who is intent on improving problem-solving will be vividly aware of this issue and may find it a comfort that it appears to be invariant with the passage of time. There never was a golden age when students were all able to read mathematical problems fluently and accurately.

Advantages of the prompt videos as a means of enhancing problem-solving skills with reference to reading comprehension and the perspectives in Table 1

There is some evidence from other fields that reading comprehension is improved when the visual stimulus is accompanied by a voice over. In the sets of videos for each past paper, each question is read out loud and an attempt is made to read the question clearly and with interest. In the prompt itself, the perspective labelled (a) is addressed by clarifying the meaning of vocabulary and reminding the viewer of the requisite concepts which have usually been encountered previously. Perspective (b) cannot be directly addressed by prompt videos. Perspective (c): it is often appropriate to indicate different solution strategies which may vary from time-consuming 'no-brain' methods to strategies which are derived from perceiving patterns. Perspective (d): metacognition is developed in part from listening to other people's commentary on their methods. Perspective (e) is addressed by providing immediate feedback when an error has occurred and allowing the viewer to complete the question themselves. Seeing one's mistake and then achieving success encourages persistence.

Conclusion: Future work and research

It is intended that this resource will find a useful place among the cornucopia of resources for learning Mathematics now available online and specifically among the resources supporting the UKMT's charitable intention and its work. The programme of work on the resource for the next year is to make sets of videos for past SMC and IMC papers, review the existing stock and raise awareness of this resource.

On the research side, installing Google Analytics will give an accurate measure of usage. It is intended to program the website to tell us how many viewers are able to answer the question correctly after one viewing of the prompt video. Other useful data may be similarly gathered.

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