"It's not worth trying to understand the problem, it doesn't help": An examination of children's approaches to solving word problems

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The research focused on strategies 29 10- and 11-year-old children applied when solving a selection of word problems. In nearly all cases children relied upon superficial, procedural strategies to identify key words and numbers that triggered inefficient and inappropriate strategies such as guess and check. Children showed remarkable resilience when applying their strategies and were confident in them even when they were unable to solve a problem. Simple interventions such as asking the children to read the problem aloud, asking prompt questions and drawing bar diagrams were decisive in helping children to understand the problem, select an appropriate strategy and reach a successful solution.

Keywords: problem solving; primary; understanding; reading

Research context

Research is undertaken not to re-create what is already known but rather to look at a familiar idea with a new lens which "makes the familiar strange, and gaps in knowledge are revealed" (Clough & Nutbrown, 2012, p.26). This study aims to contribute to the knowledge of word problem-solving by examining the strategies children used and their explanations. Solving word problems is complex. Effective problem solvers have to translate word problems into a visual model of the problem, affected by semantics and language (Hegarty & Kozhevnikov, 1999; Cummins, 1991). The order of the knowns and unknowns (Littlefield & Rieser, 1993; Thevenot, 2007; Cook, 2011), the complexity of information processing required (Ilany & Margolin, 2010), whether problem information is explicit, the complexity of the mathematical idea captured in the problem (Greer, 1997) and its structure (Kintsch & Greeno, 1985) all contribute to a problem's level of difficulty. Children may not recognise how different parts of a problem are connected or that solving problems is an iterative process (Ilany & Margolin, 2010; Voyer, 2010). Children may not have read the problem properly (Fuchs et al., 2020) or relate to the context (Gerofsky, 1996). Furthermore, children's attitudes can also affect how they solve problems. They may not expect to understand the problem if doing so is not encouraged by their teachers (Sepeng & Webb, 2012; Pakarinen & Kikas, 2019). Similarly, their problem-solving strategies may have been shaped by teachers who lack confidence in problem-solving themselves and have limited teaching methods (Leikin & Levav-Waynberg, 2007).

The use of inappropriate problem-solving strategies occurs when children do not understand a problem. Unsuccessful problem solvers focus on the surface features of a problem (Silver, 2013). Consequently, superficial strategies such as direct translation (also known as number grabbing), where students perform calculations based on numbers selected from the problem, are often applied (Mayer, et al., 1996; Rouet, 2006). This phenomenon is termed "compute first and think later" (Stigler, et al., 1990, p.15). Other unsuccessful problem solvers latch on to keywords which is unhelpful when the words "generally associated with a given operation provide a crutch upon which pupils may come to rely" (Carpenter, et al., 1980, p.11) For example, words or phrases such as 'more than' or 'altogether' would trigger an addition calculation (Carpenter, et al., 1980; Hegarty, et al., 1995; Thevenot, et al., 2007). Relying on trigger words is particularly prevalent if children have found it to be a generally helpful strategy (Bruner, 1977). However, it is error prone as the word 'share' could commonly relate to a division situation (Tom shared 6 apples between 3 friends) or a subtraction situation (Tom shared his 6 apples with 3 friends. After he gave them each one apple, how many apples did Tom have left?).

Carpenter, et al. (1993, p.429) found that frequently children seemed to ignore the obvious features of a problem; instead, they adopt a "mechanical application of arithmetic and algebraic skills". Thevenot (2017, p.57) cautions that "an automatic application of these schemata can undermine performance and can sometimes prevent individuals from setting up more efficient and less cognitively demanding strategies". Bruner (1977) and Boesen (2010) found that once children think that they have a routine to solve a problem, they are reluctant to change strategy, even if it is inappropriate. However, reaching a successful solution requires a child to start again, but they would only do so if they realised that they were wrong and had an alternative 'bright idea' (Mason, et al., 2010). Mason, et al. (2010, p.30) state that "the victim is usually unaware that this is the root of the trouble". Consequently, children may "jump at the first idea which comes along" they "attack" a problem without really considering the best way forward.

Research approach

As problem-solving is so complex, it is important to understand what children do when presented with an unfamiliar problem. I was therefore interested in researching children's strategies to identify if some generalisable themes might emerge to help improve children as problem solvers. My research, therefore, sought to examine which strategies children used and focused on the following questions:

- What did children do to help them understand the problem?
- What strategies did children use when trying to solve the problem?
- What did children do when they were stuck?

The research took place in the summer term of 2017. It involved 29 10- and 11-year-old children from nine schools across three local authorities. The participants were selected by the schools and were asked to identify children who were working at age related expectations, had no barriers to learning mathematics and would be willing to work with an unfamiliar adult. Following Plowright's (2012) mixed-methods approach, my research involved observation, asking questions and artefact analysis.

Research interviews comprised up to four phases. The first phase involved children being observed whilst sorting a selection of word problems from easiest to hardest (Figure 1). Children were also observed during the second phase, where they attempted to solve problems. I acted as an overt, non-participant observer during these phases. Once a problem was completed, the third phase commenced as I questioned them about their solution. If their solution was appropriate, children chose a further problem, and the phases were repeated. However, if a child became stuck or had reached an incorrect solution, a fourth phase began as I moved out of the researcher role and into a participant-observer role and intervened in the solutions to prompt a deeper understanding through asking questions and drawing bar models. Once a correct solution had been reached, I reverted to overt non-participant observer as the child attempted to solve a further problem. Each interview was video recorded, and field notes were taken.

KEENAN: In Keenan's toy bin there are 24 red blocks. There are 13 more yellow blocks than red blocks. There are also 14 more blue blocks than red blocks. How many blocks are there in all?
LISA: Lisa is practicing addition and subtraction problems. What number should Lisa add to 142 to get 369?
FAIR: A man took his 3 children to a Fair. Tickets cost twice as much for adults as for a child. The father paid a total of £50 for the 4 tickets. How much did each child's ticket cost?
TIM: Tim gives £655 to two charities. He shares it so Cat Rescue gets 4 times as much as the Home for Stray Dogs. How much does each charity receive?
TINA: Tina had twice as many game cards as Kevin. Nick had twice as many game cards as Tina. The 3 pupils had 357 game cards altogether How many game cards did Kevin have? How many game cards must Nick give to Tina and Kevin separately so that all 3 of them have number of game cards?

Figure 1: Sample of word problems used

Research findings

Children's responses suggested that their judgements of problem complexity were based on a superficial reading of the problems (Figure 2).

What makes a problem hard or easy?	
A problem is hard if ….	A problem is easy if …
'I don't like percentages'	'it's just adding'
'it's really long'	'I already know the answer'
'I'm not very good at multiplication'	'if I can do it in my head straight away it's easy'
'I don't know really, I just look at it and think that looks easy or hard'	'I just start putting in a start number, like 12 … and see if it works'
'I'm all rush, rush so I didn't read the question properly'	

Figure 2: Why a problem might be hard

Their responses suggested an hypothesis.

Hypothesis: Children do not read problems to understand them.

Children's responses (n=19) to Keenan's problem (Figure 3) were analysed to consider this hypothesis.

In Keenan's toy bin there are 24 red blocks. There are 13 more yellow blocks than red blocks. There are also 14 more blue blocks than red blocks. How many blocks are there in all?

Figure 3: Keenan's problem

This is a routine addition problem. There are only explicit propositions, no distractors and the order of the knowns and unknowns largely match the order of the numbers as they are written in the problem. However, only 11 solved the problem without assistance. Six children required intervention and two children did not solve the problem (they ran out of time to do so). The most frequent incorrect solution was a direct calculation strategy where children added the numbers in order (24 + 13 + 14 = 51) (Cha, et al., 2007; Goodstein, et al., 1971). Their confidence in their solutions suggest that they matched their interpretation to the solution (Xin, 2007) rather than reading the problem to understand it. Consequently, once the solution had been triggered, each step reinforced their solution, and children did not seek alternative clues (Bruner, 1976; Thevenot, et al., 2007; Boesen, et al., 2010), so there was no strategy modification. Questioning the children about their strategies revealed that they had not attempted to understand the problem but rather that they were influenced by superficial features. Some children commented that they "just looked for numbers". One child said that this was because "it looked really confusing because … there's all stuff in just one question. It just makes me confused". Another child was asked if reading the problem properly before attempting to solve it would be helpful. He responded, "it's not worth trying to understand the problem, it doesn't help". It is noteworthy that the use of procedural strategies was extensive, and direct calculation solutions were seen across all schools. The ubiquitous use of direct calculation and children's responses indicates that children did not attempt to understand a problem before solving it and were willing to apply inappropriate solutions.

Interventions

If children became stuck when solving a problem, they either persevered with their inappropriate strategy or stopped trying to solve the problem. At these points I introduced an intervention. The interventions were based on Mason, et al.'s (2010) model. The first intervention was to ask the child to read the problem aloud. For six children who had reached incorrect solutions, reading the question aloud enabled them to notice the relationship between the elements of the problem and its iterative nature and solve the problem correctly. For three further children asking them 'what do you know?' and 'what do you want to find out?' 'so what will you try?' was sufficient to enable the child to reach a correct solution. These simple interventions were effective across all schools. For multiplicative reasoning problems, the previous strategies were combined with drawing bar models to enable the children to sufficiently understand the problem to reach an appropriate solution.

Why this study matters

The extensive use of inefficient number grabbing across all nine schools indicates how widespread inefficient problem-solving strategies are. The implications of their use are potentially significant. It took 11 seconds for some children to solve Keenan, but the slowest solution was 11 minutes. Those who use number grabbing showed great confidence and resilience and were happy to be busy problem solvers. However, they did not recognise the fault in their solutions and would not have noticed that their solutions were unreasonable without intervention. Children's inability to detect inappropriate solutions without prompting was even more noticeable on more complex problems. As long as children are tested on solving problems encouraging them to read them aloud and respond to 'what do you know?' what do you want?' and 'what will you try?' strategies appear to be helpful strategies in increasing efficiency

Reflections

Although the interventions were most helpful in enabling children to be successful problem solvers, this research raised other questions. It was apparent that children did not consider understanding the problem to be of any relevance but valued resilience and perseverance more. It would be interesting to consider further why this might be the case and whether children's and teachers' views could be sought to explore this issue further. Another area that could be examined is why there is such dependence on simple strategies such as direct calculation, and in the case of multiplicative reasoning problems, guess and check and the extent to which this reflects what is taught. However, these questions presume that solving word problems is inherently valuable. The scenarios and ways they are expressed are usually implausible, the phrasing manipulated to create an "interesting" problem. However, they are not 'real life; if Keenan had wanted to know how many blocks he had, he could have counted them. If the purpose of solving word problems is to develop algebraic thinking, perhaps instead of focusing on word problem-solving strategies, more time could be spent on teaching algebra in more accessible ways.

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